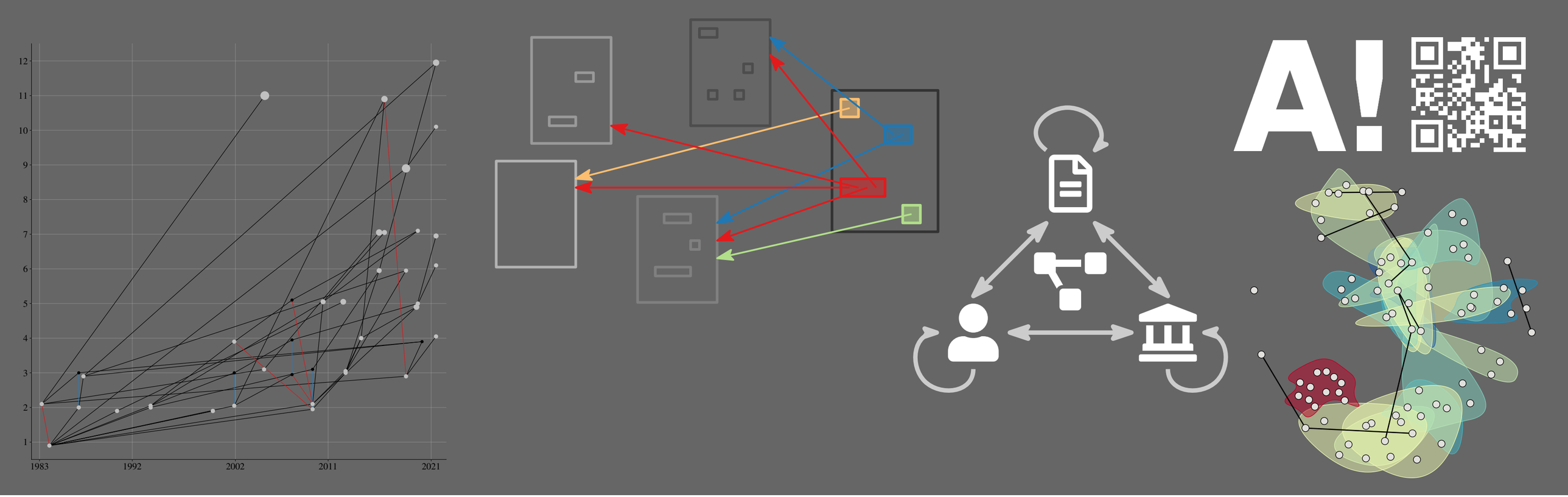
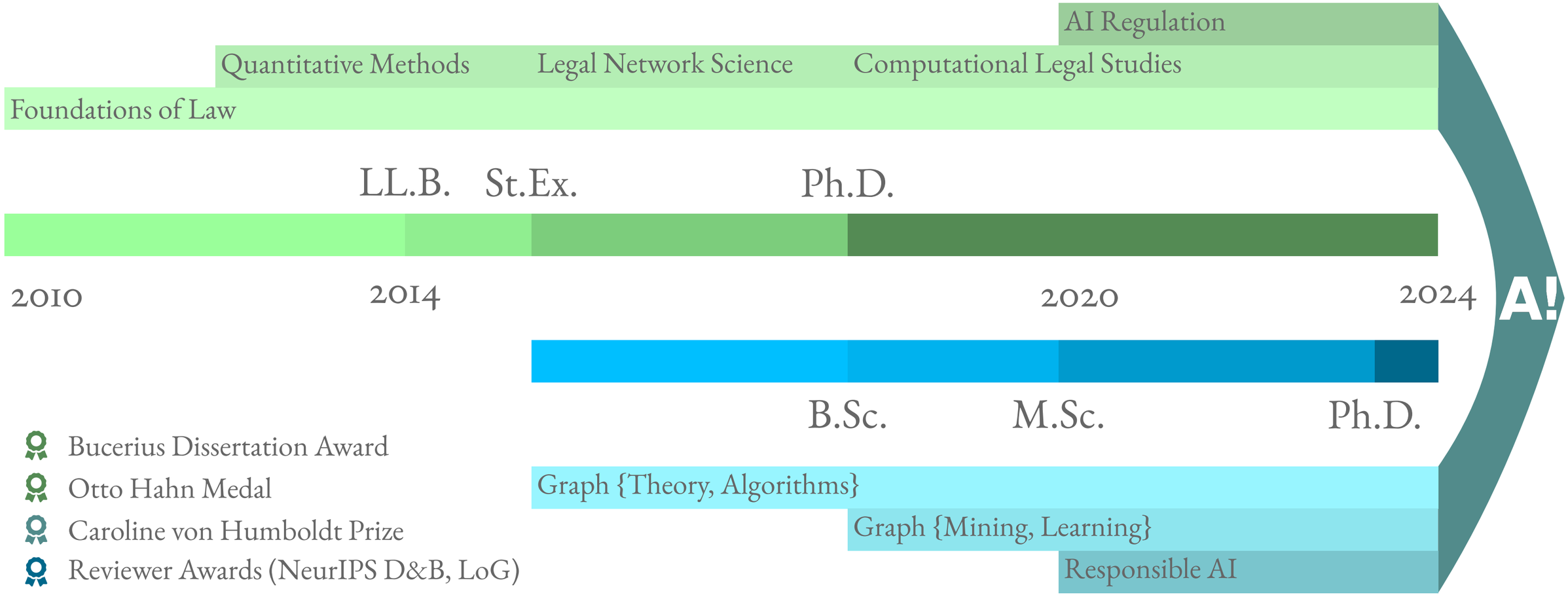


The Anatomy of Legal Argumentation

Harnessing Higher-Order Networks to Trace the Development of Doctrine

Corinna Coupette

Scientific Trajectory: Internalizing Interdisciplinarity



- Bucerius Dissertation Award
- Otto Hahn Medal
- Caroline von Humboldt Prize
- Reviewer Awards (NeurIPS D&B, LoG)
- Digital Futures Postdoctoral Fellowship

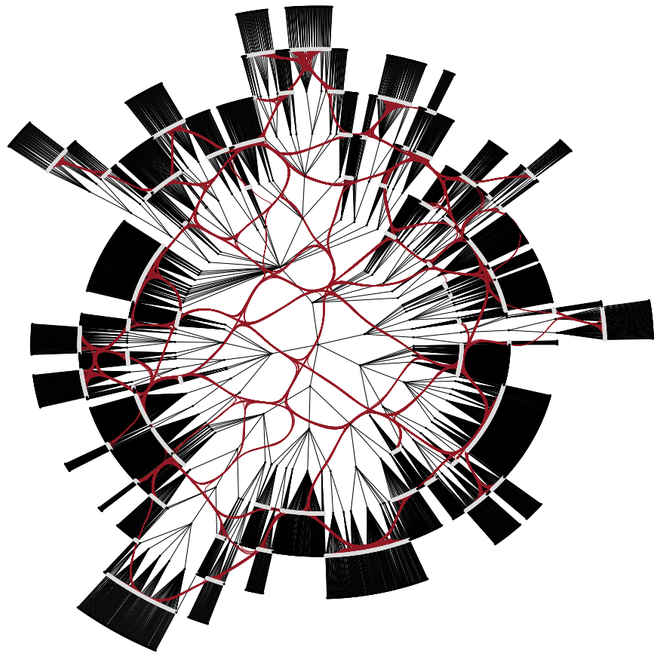
The Telos Lab: Technology and Law for Society

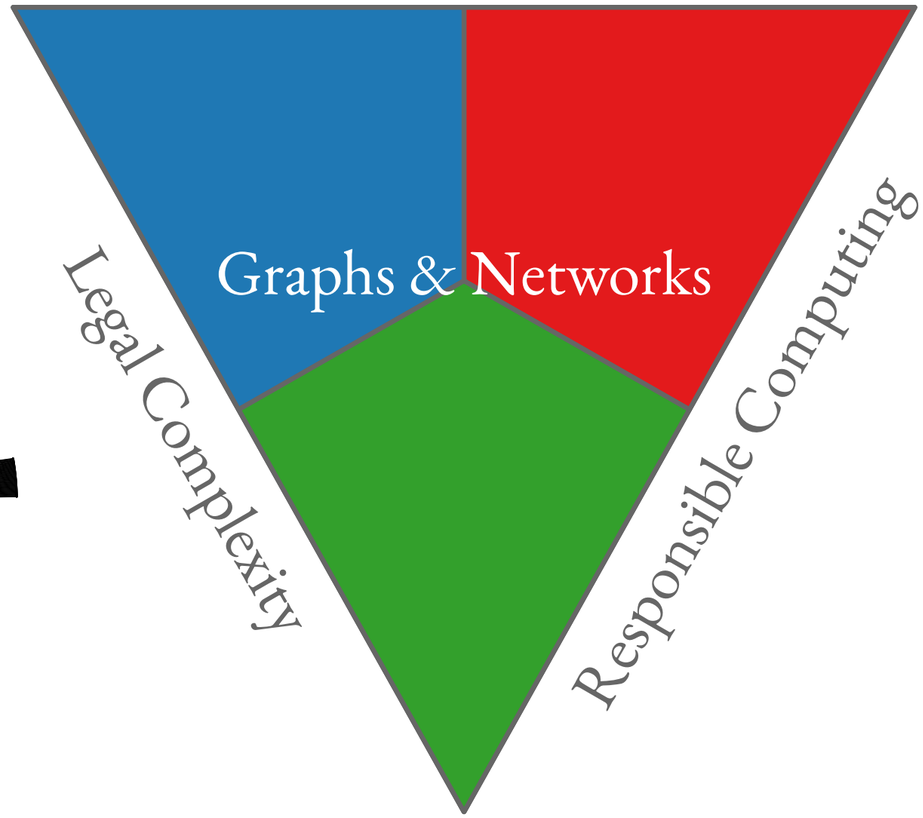
τέλος = Purpose

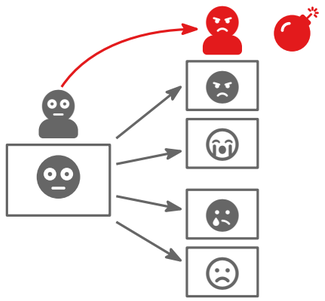
Law

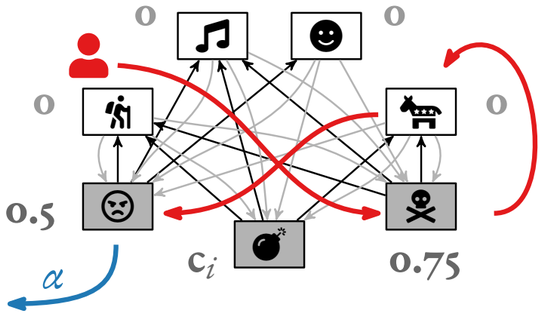
Computational Justice

Computer Science









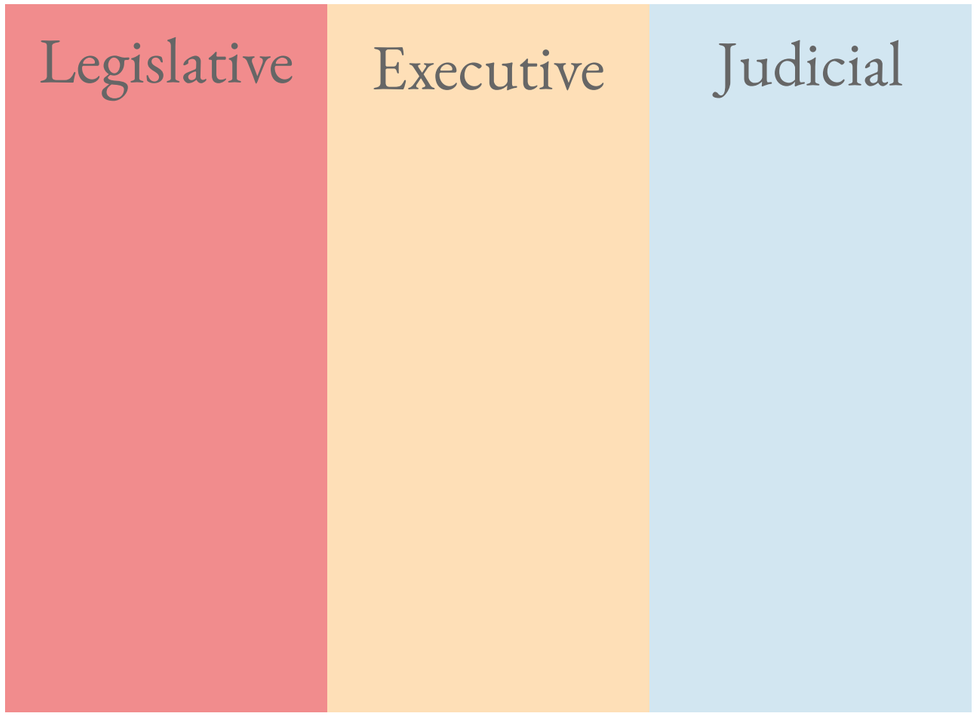
Complex Systems

Part I

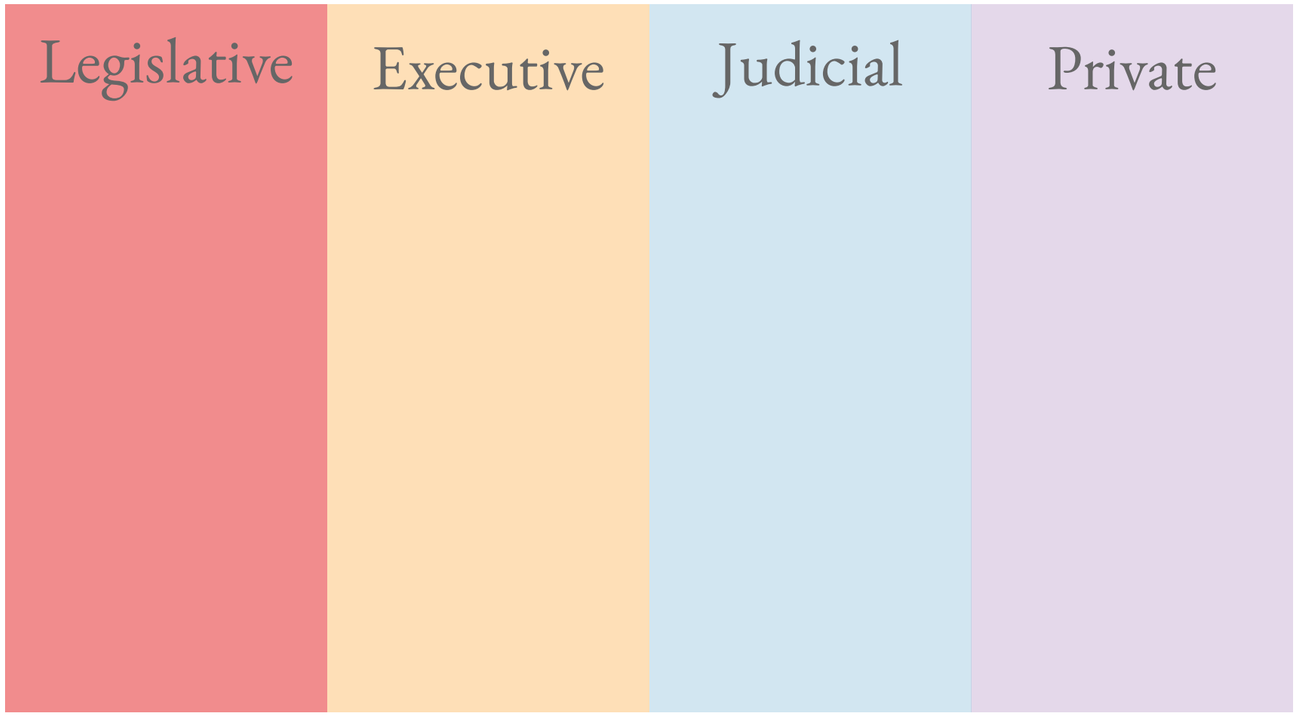
A Primer on Legal Network Analysis

Legal Systems

Legal Systems



Legal Systems



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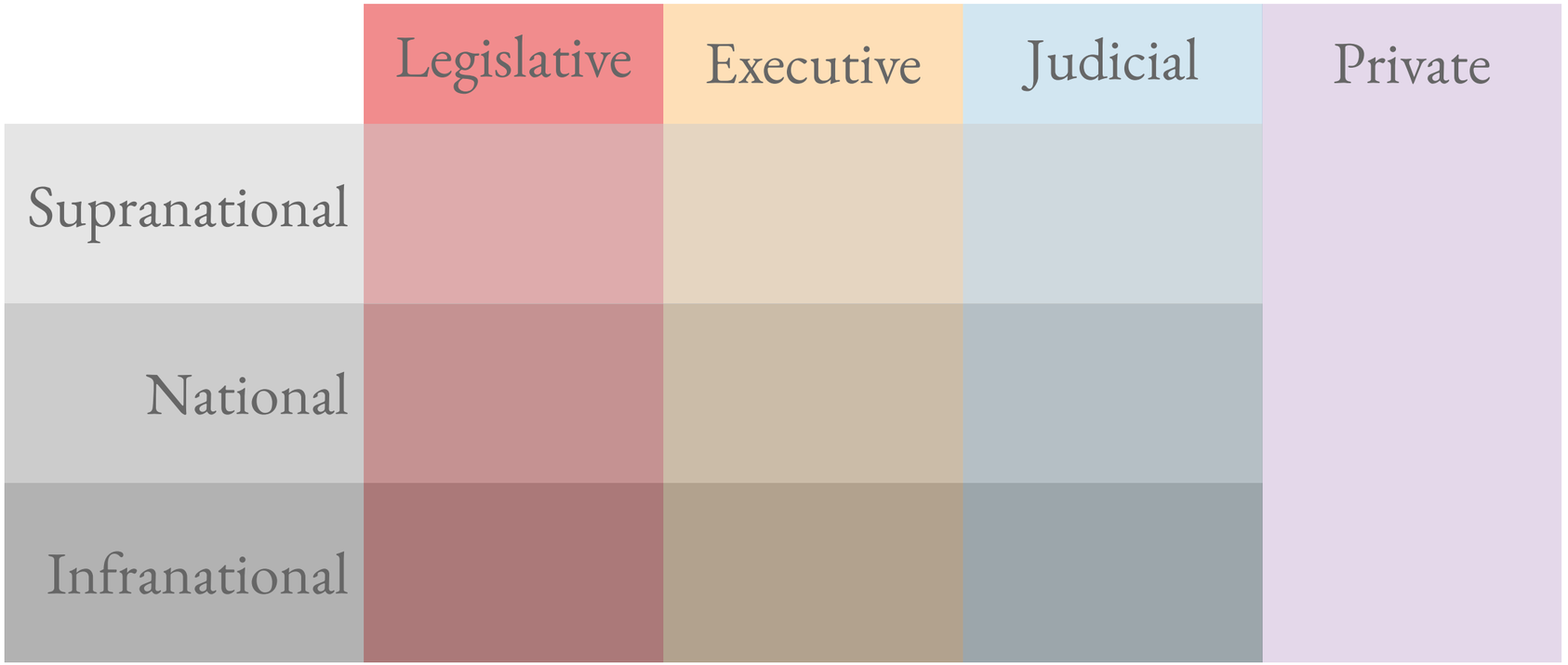

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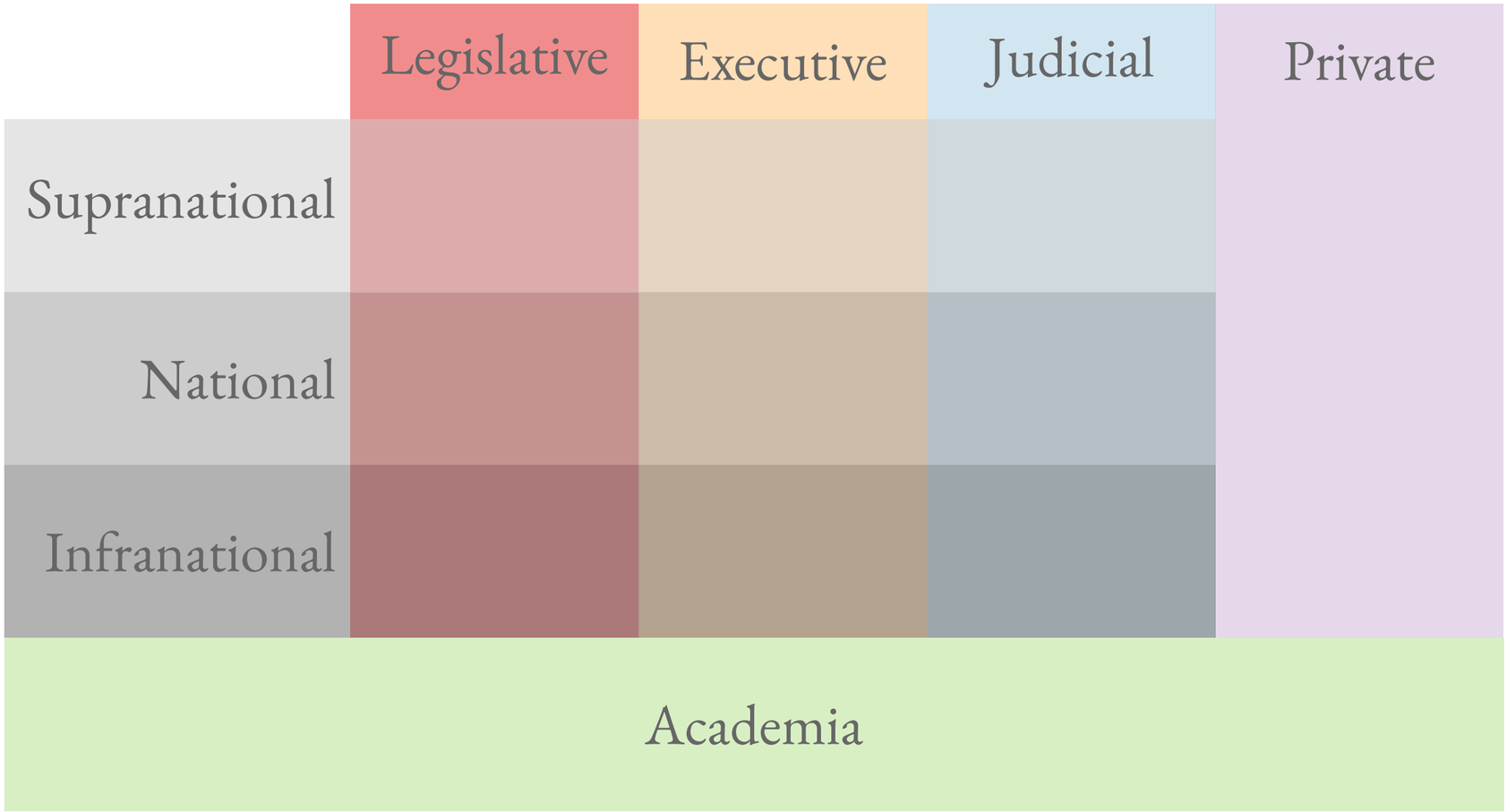

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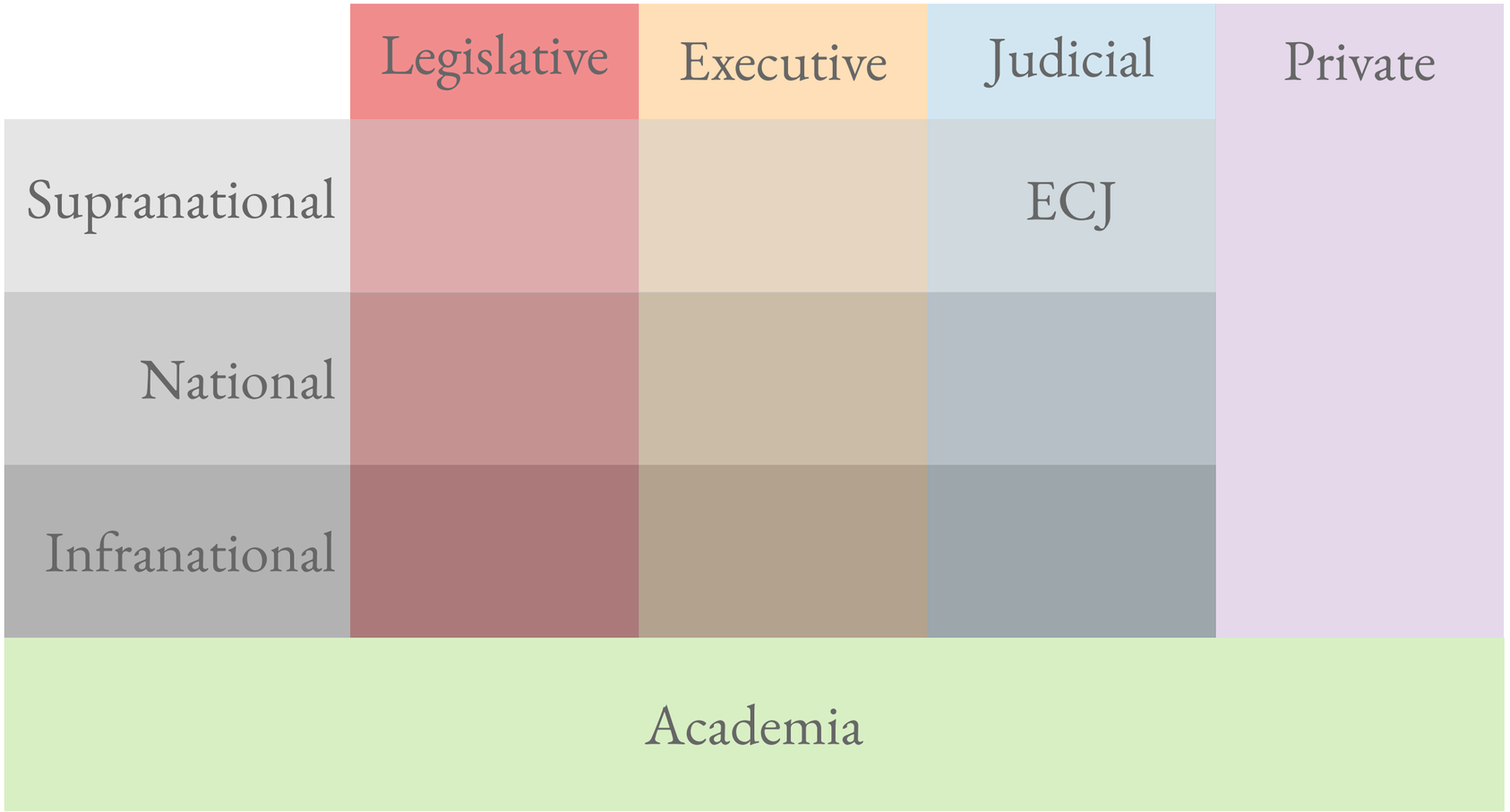

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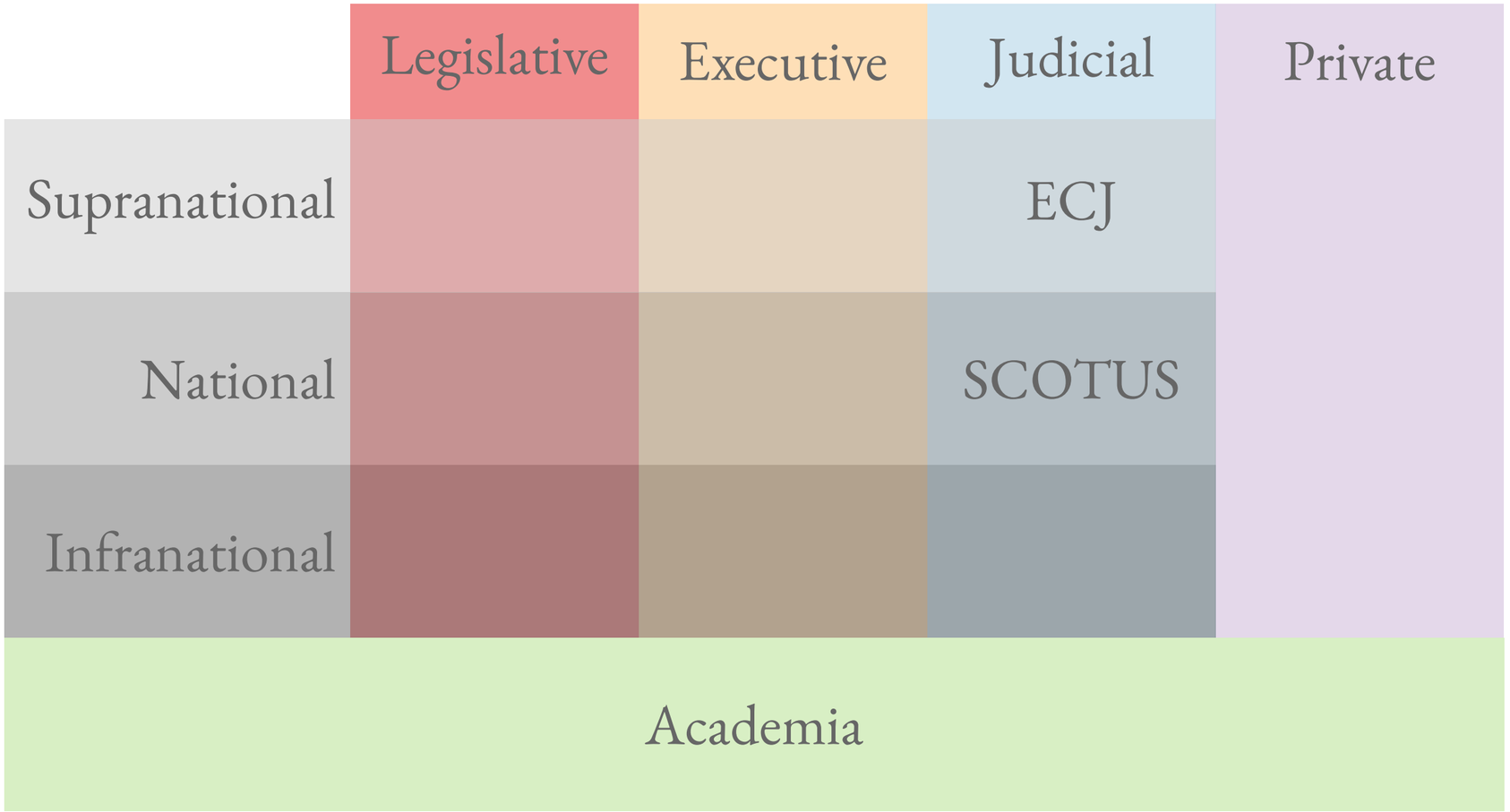
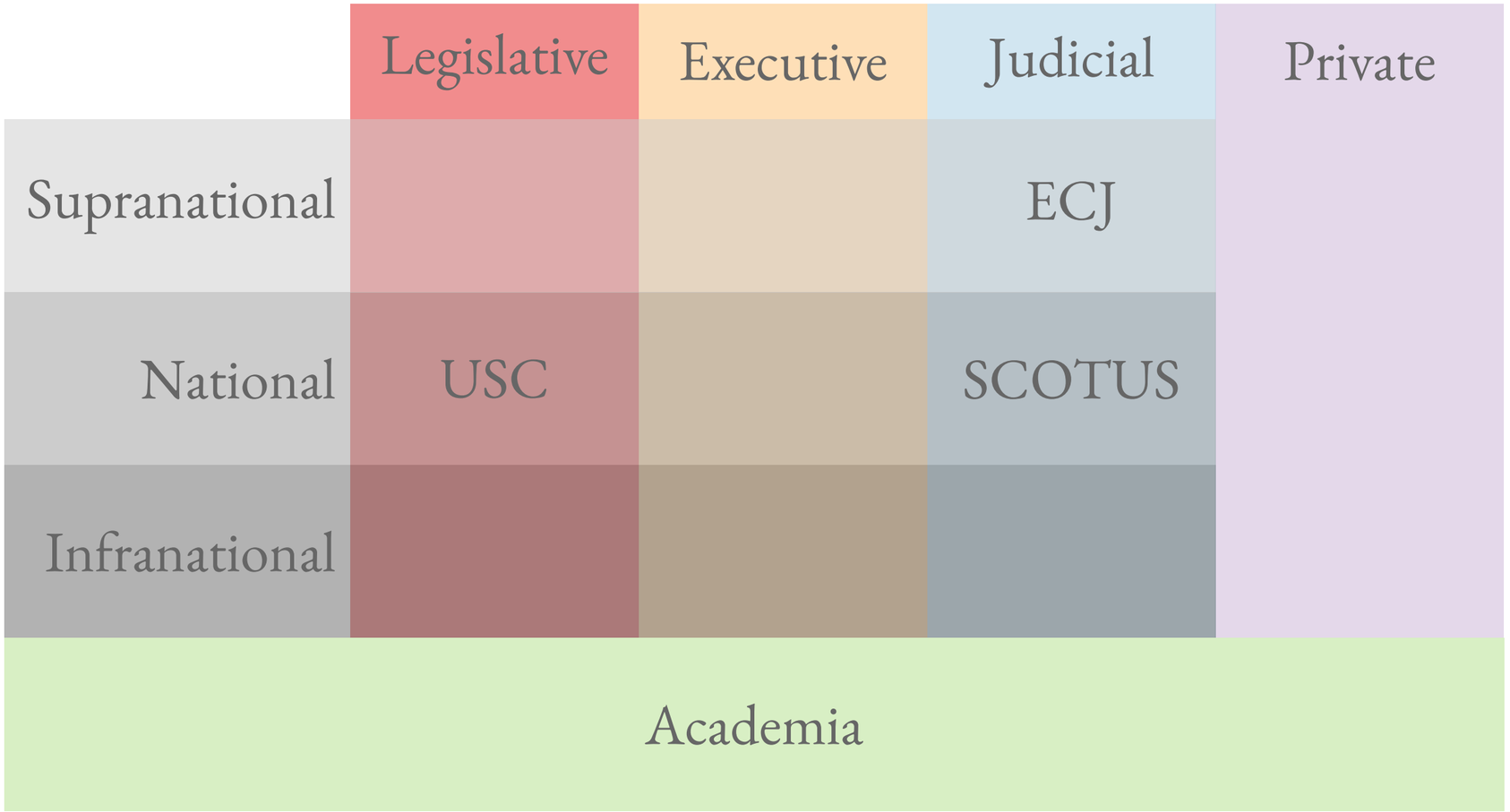


Legal Systems



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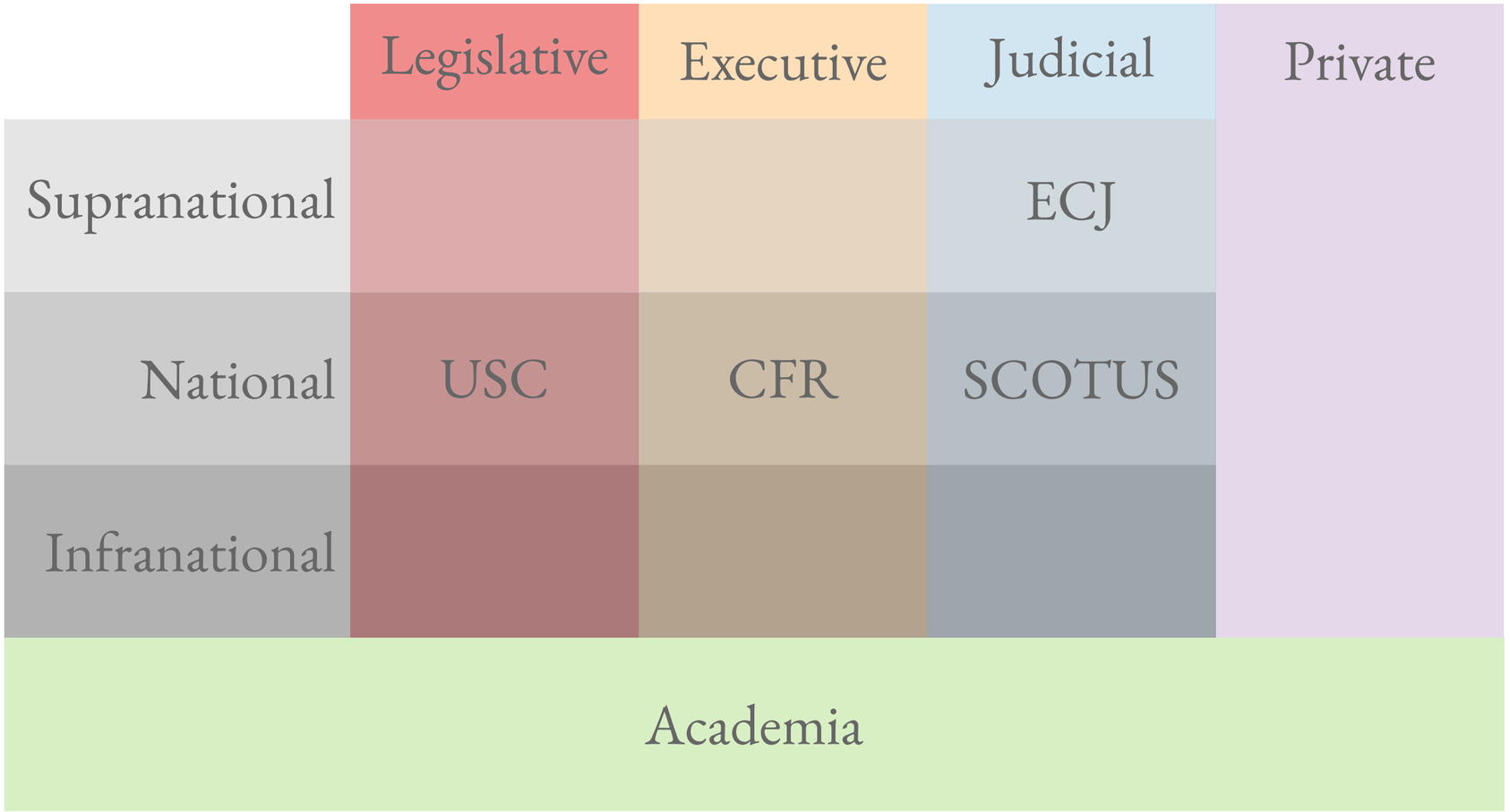

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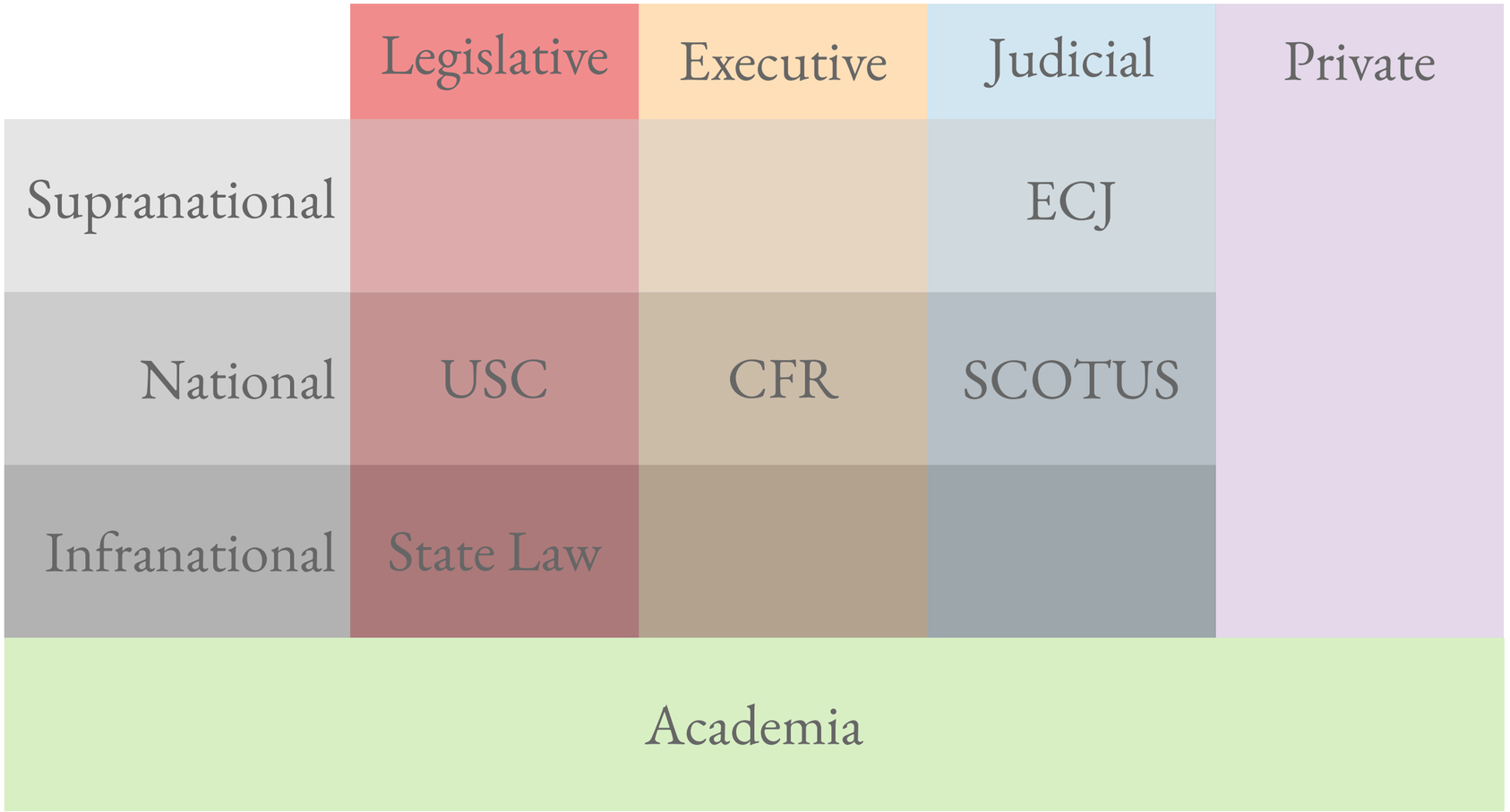

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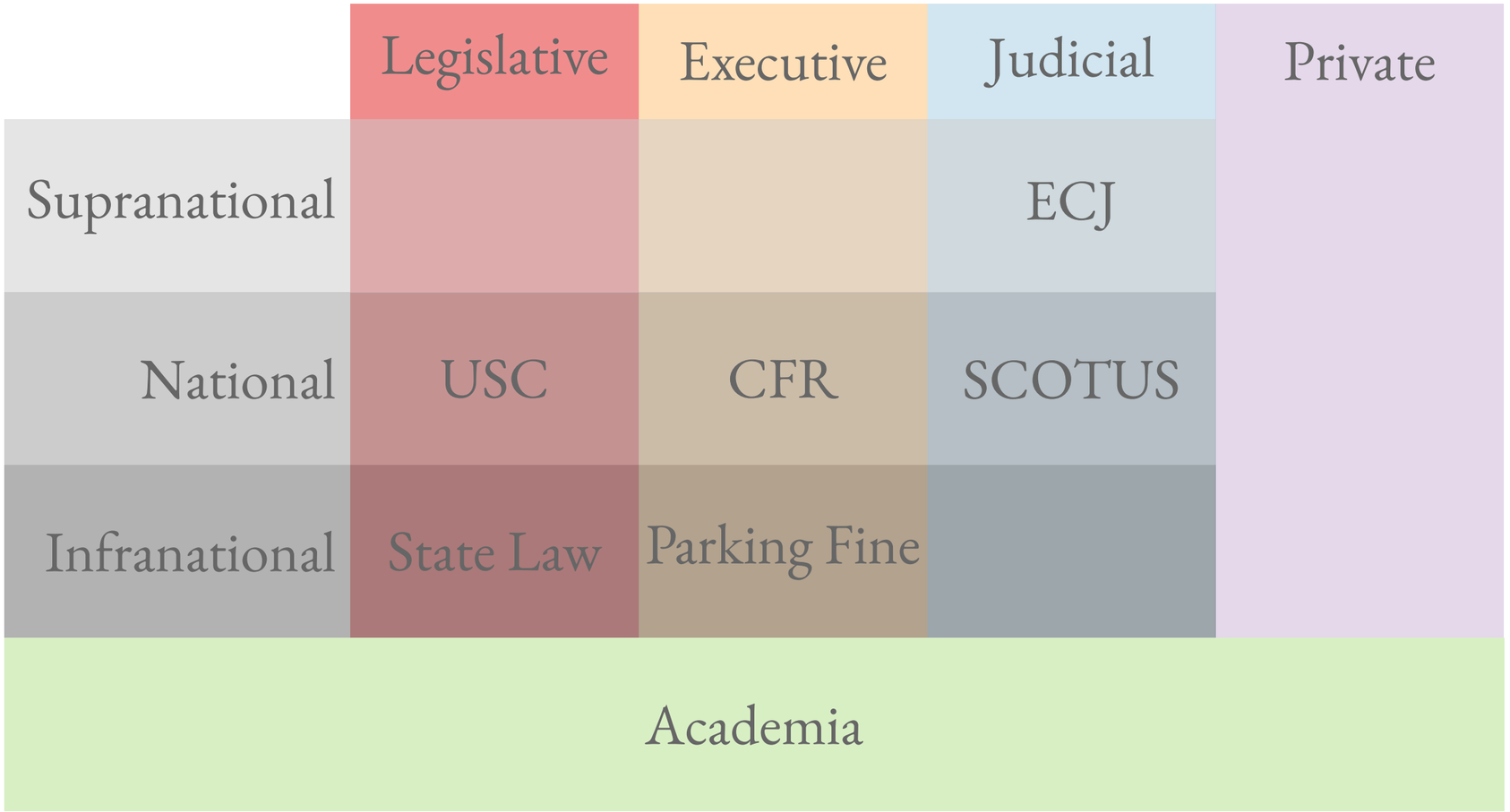

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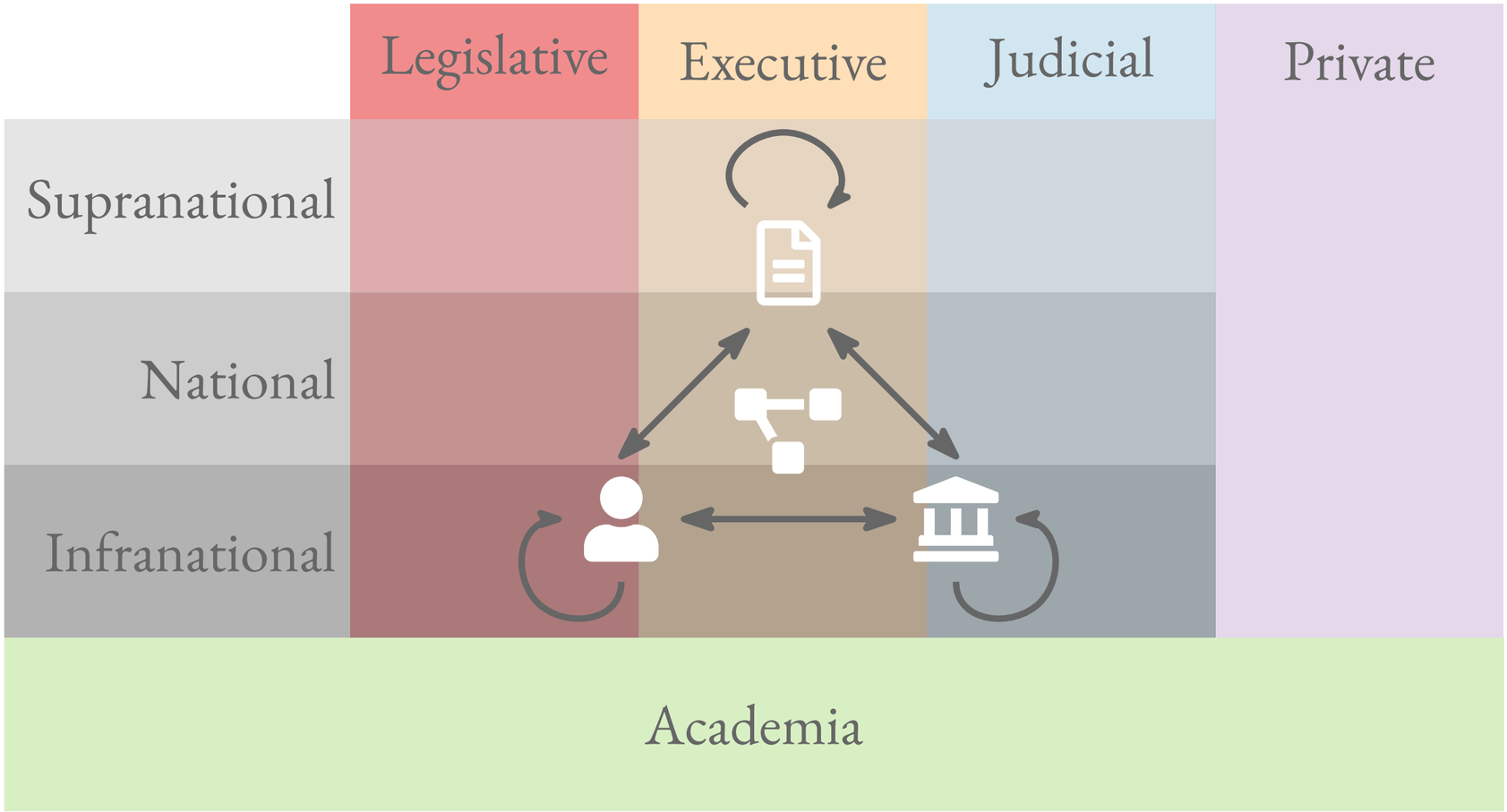

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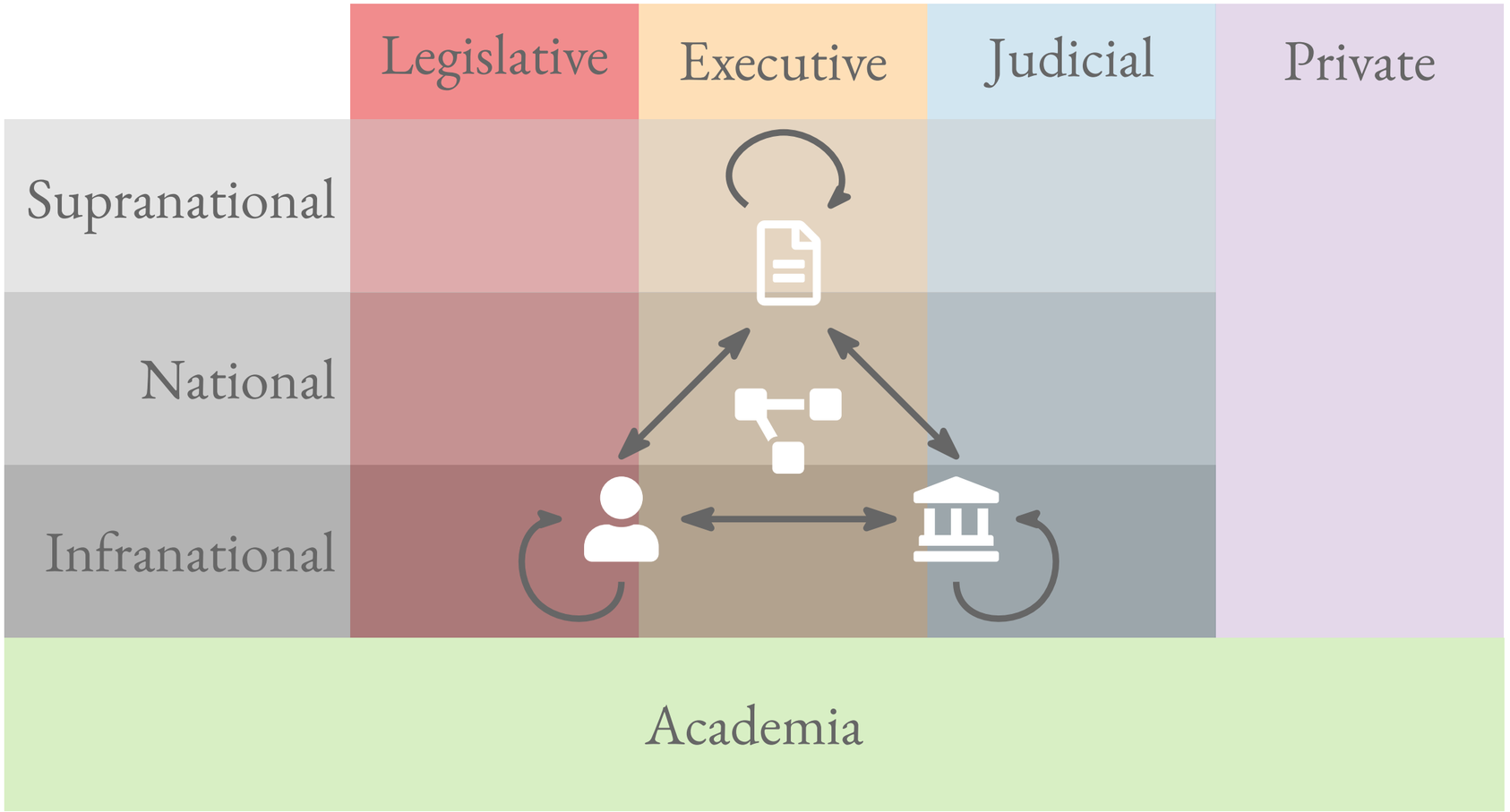

Data

Models

Methods

From Legal Systems to Networks and Graphs

From Legal Systems to Networks and Graphs

Legal systems are *complex systems*.

From Legal Systems to Networks and Graphs

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Complex systems are naturally modeled as *networks*.

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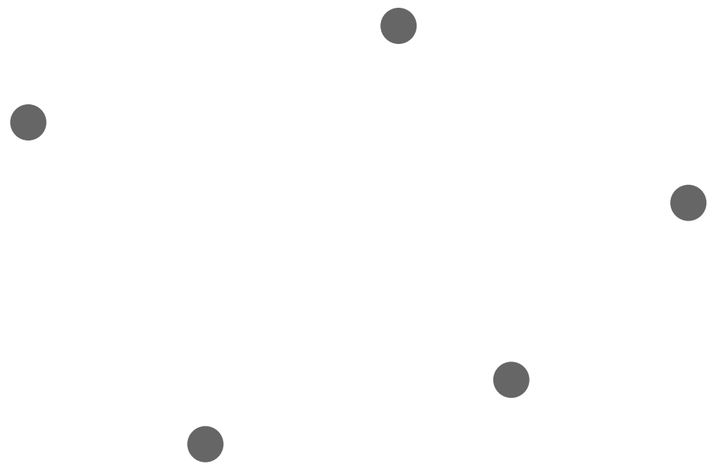

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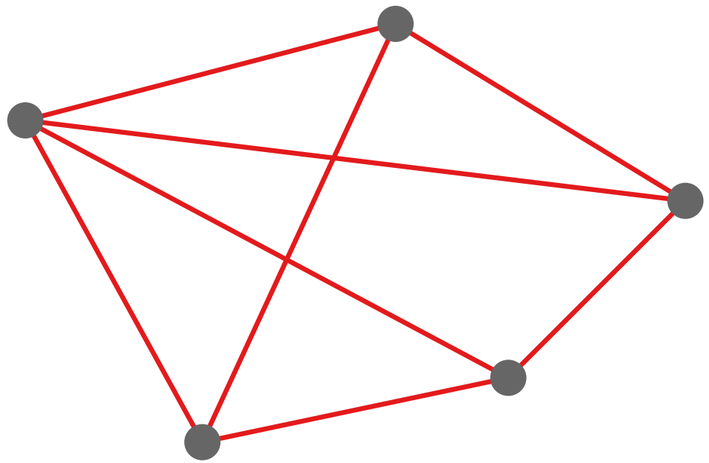

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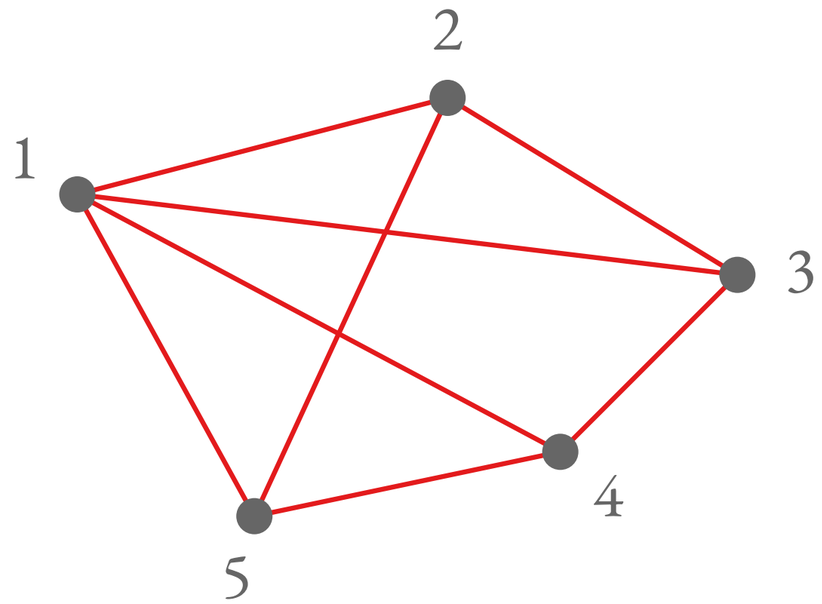

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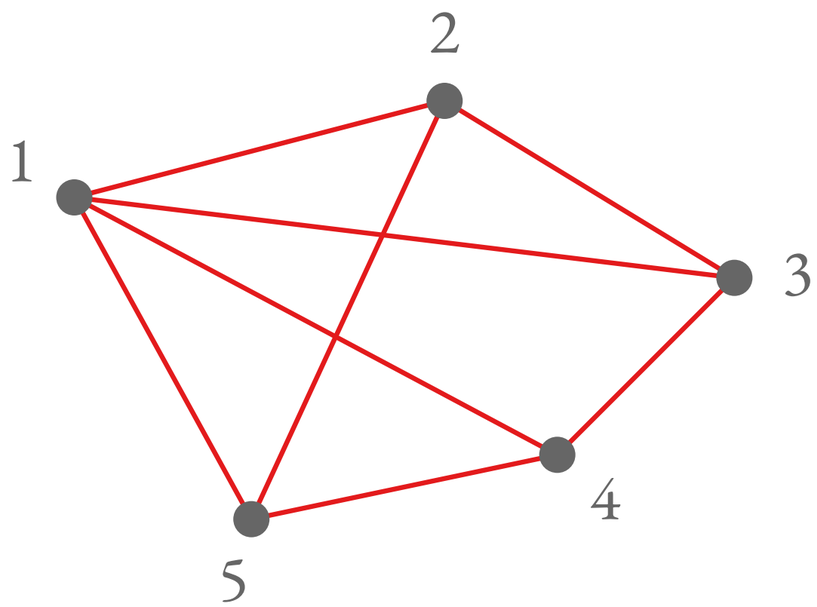

Graph G

$\Leftrightarrow$

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Adjacency Matrix A

From Legal Systems to Networks and Graphs

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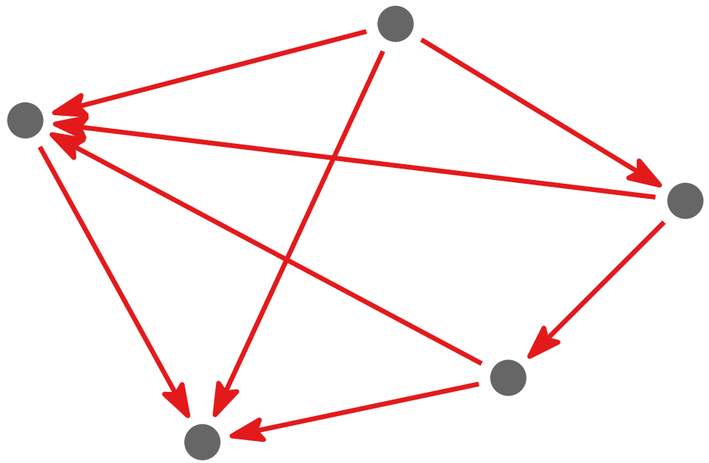
- Edges E



Directed Graph

From Legal Systems to Networks and Graphs

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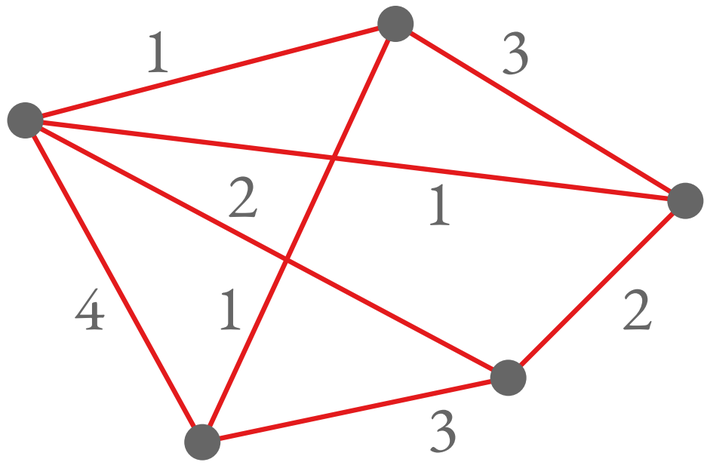

Weighted Graph

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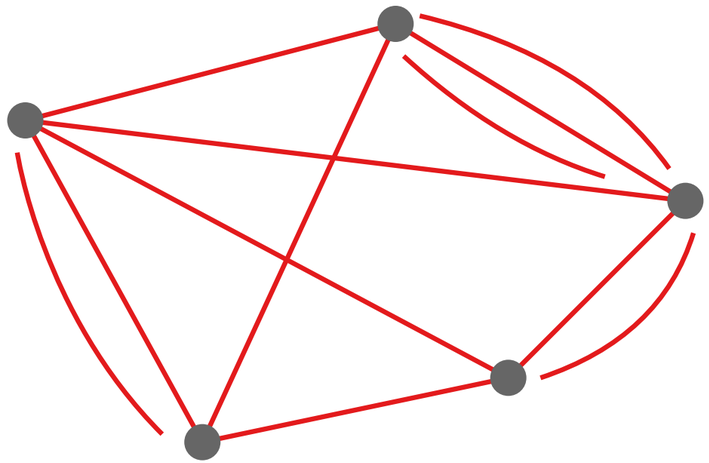

Multigraph

From Legal Systems to Networks and Graphs

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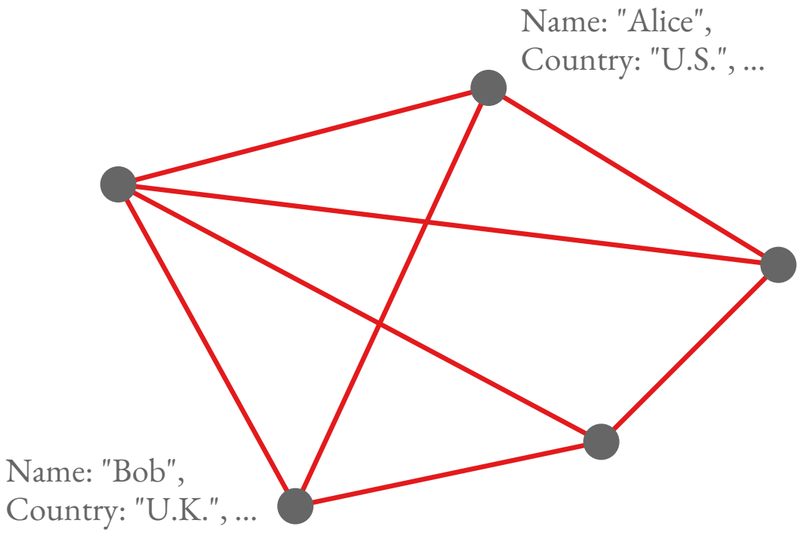
- Edges E



Attributed Graph

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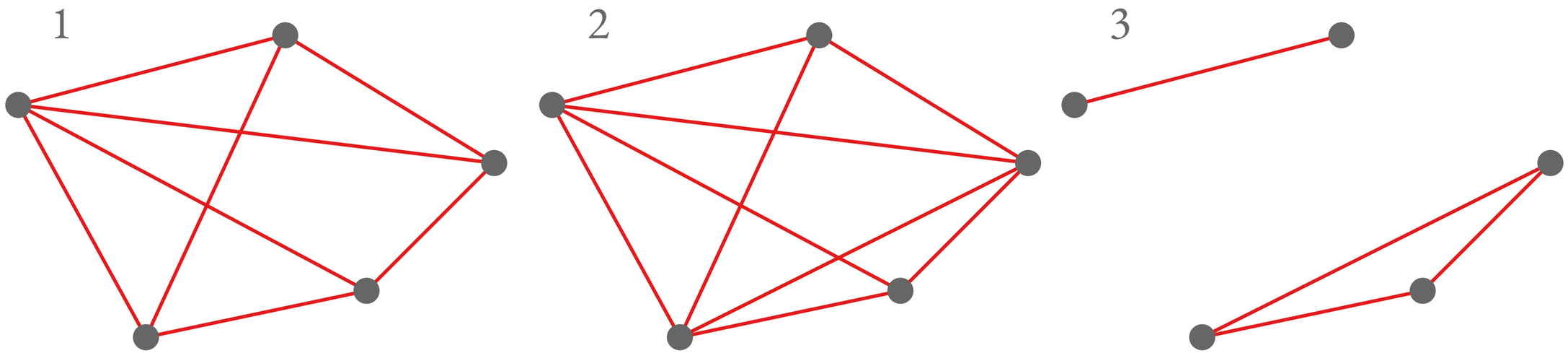

Temporal Graph

From Legal Systems to Networks and Graphs

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- Nodes V
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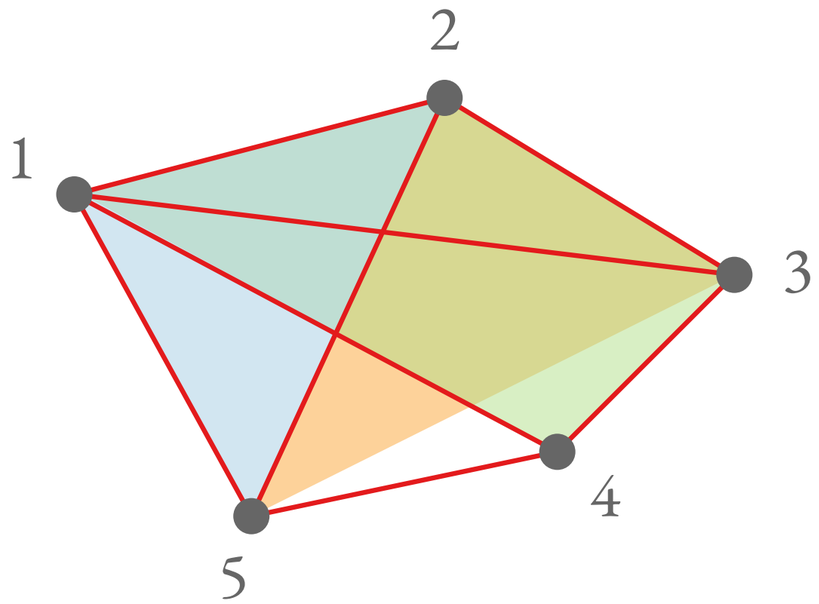


Hypergraph

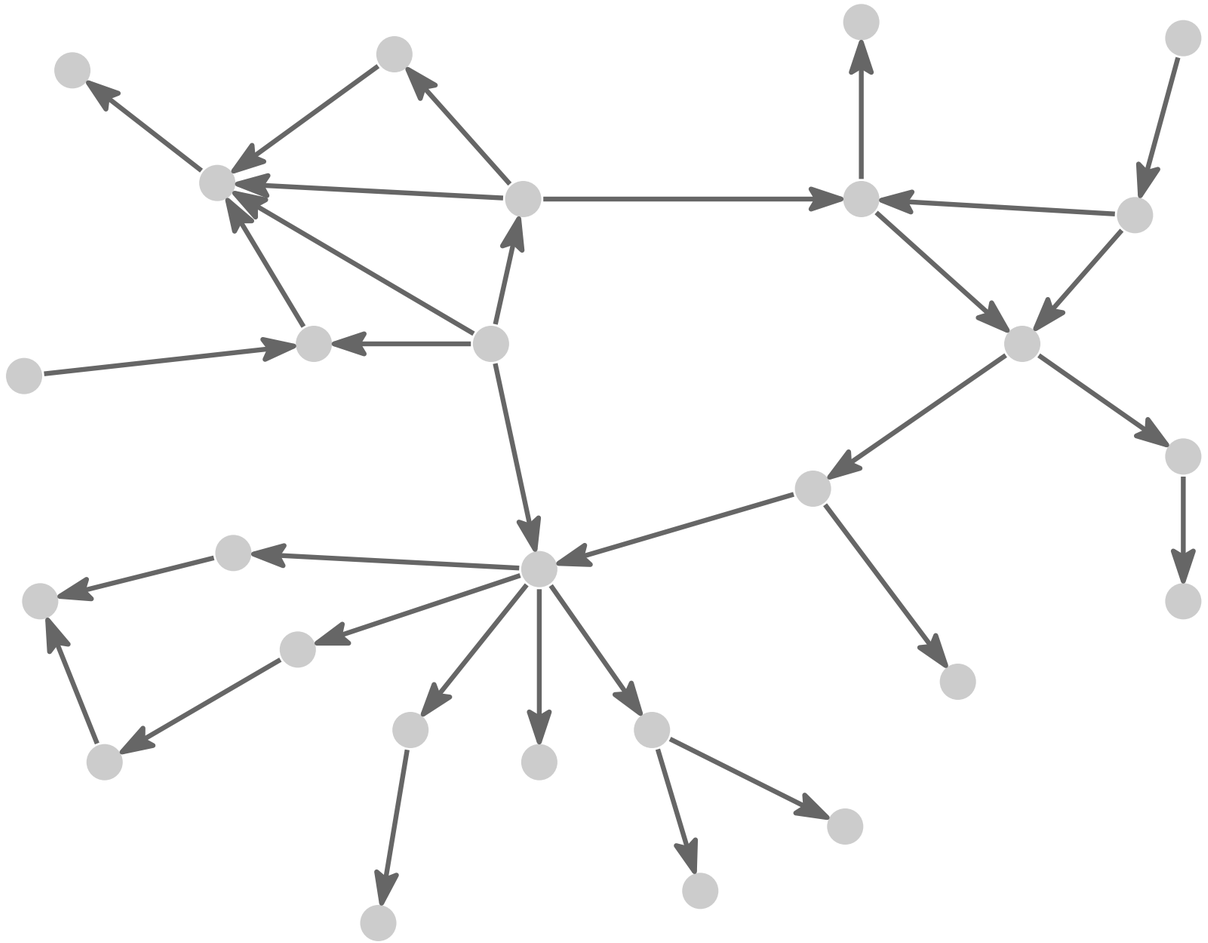
$h_1 = \{1, 2, 5\}$

$h_2 = \{1, 3, 4, 5\}$

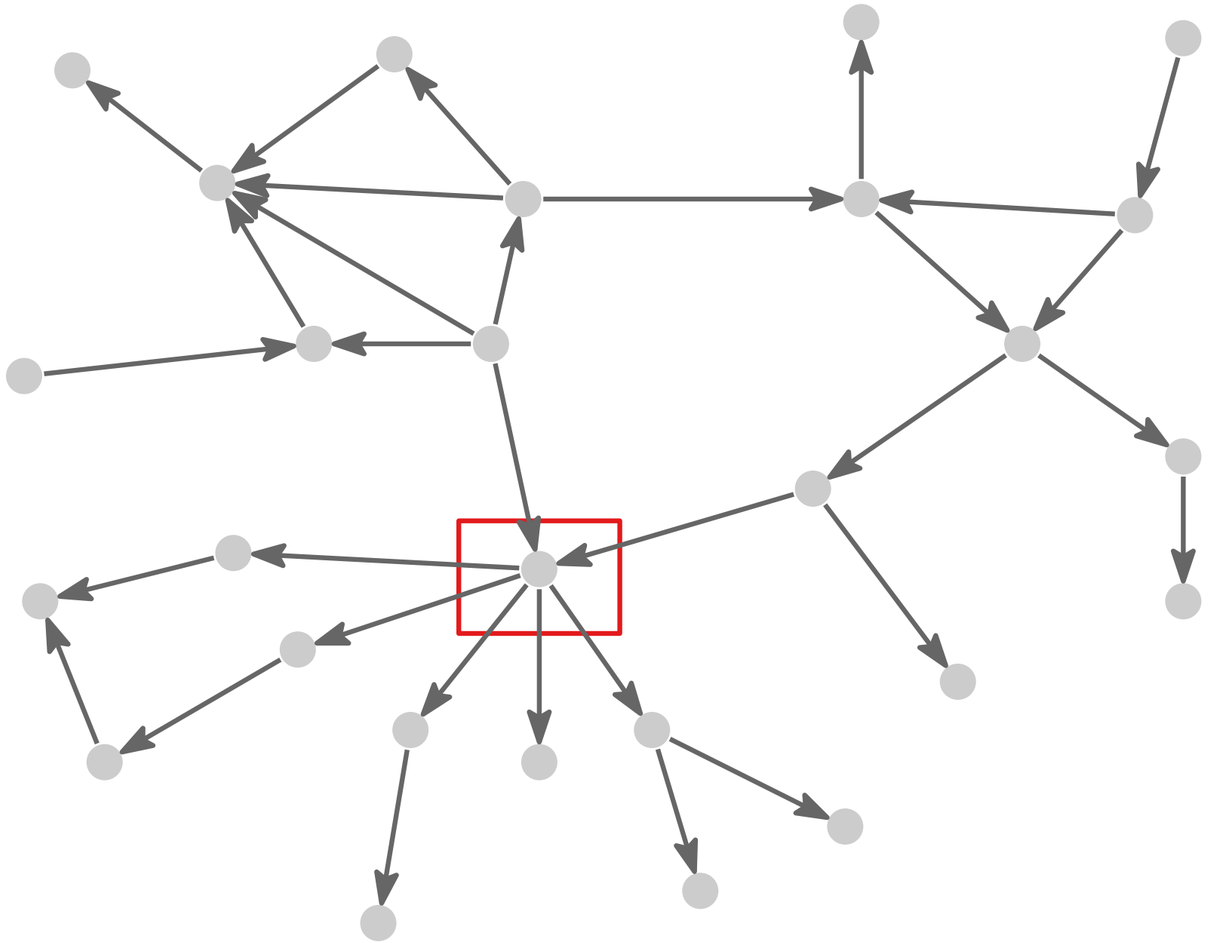
$h_3 = \{1, 3, 5\}$

Basic Levels of Analysis

Basic Levels of Analysis



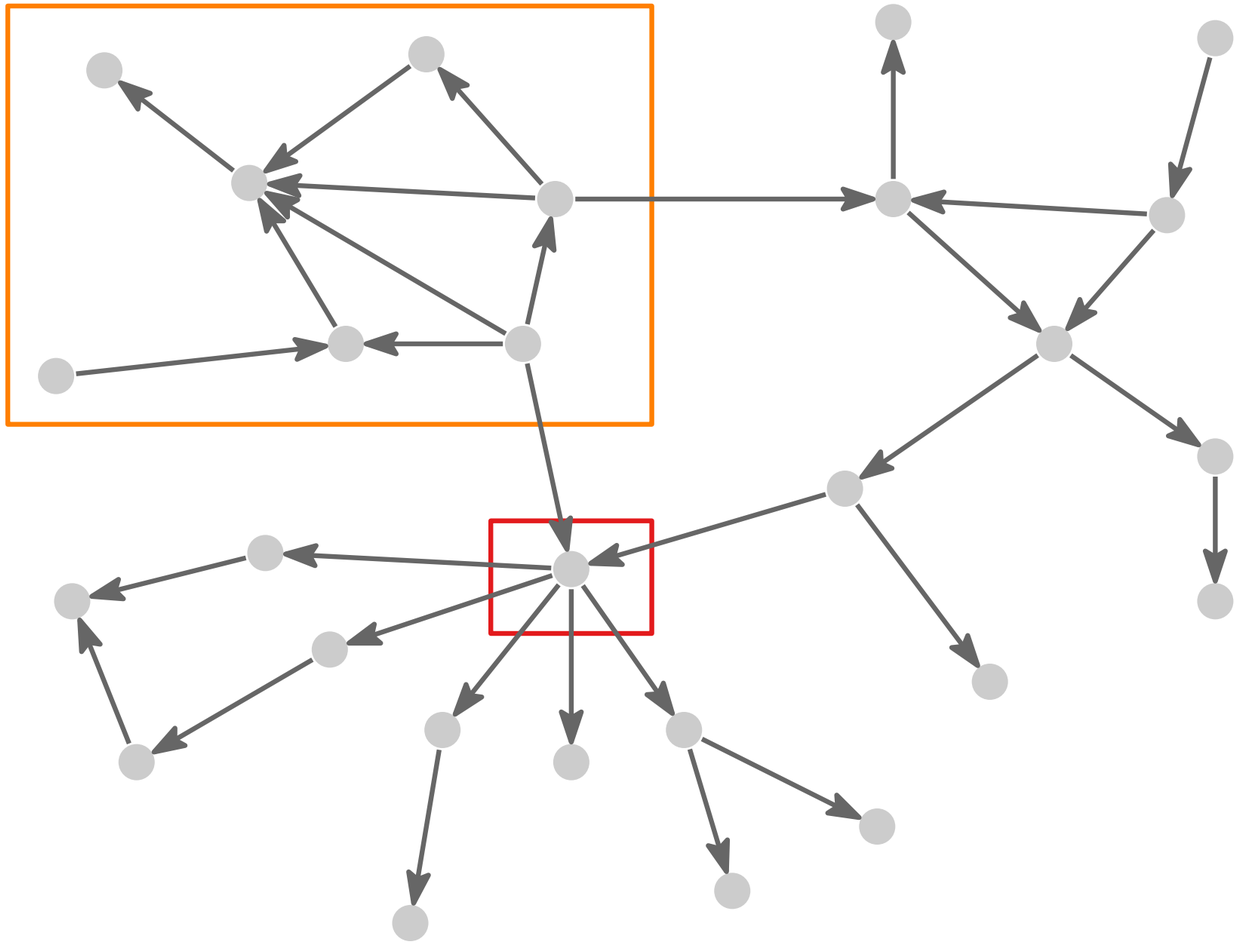
Basic Levels of Analysis



Micro Level

Nodes & Neighborhoods

Basic Levels of Analysis



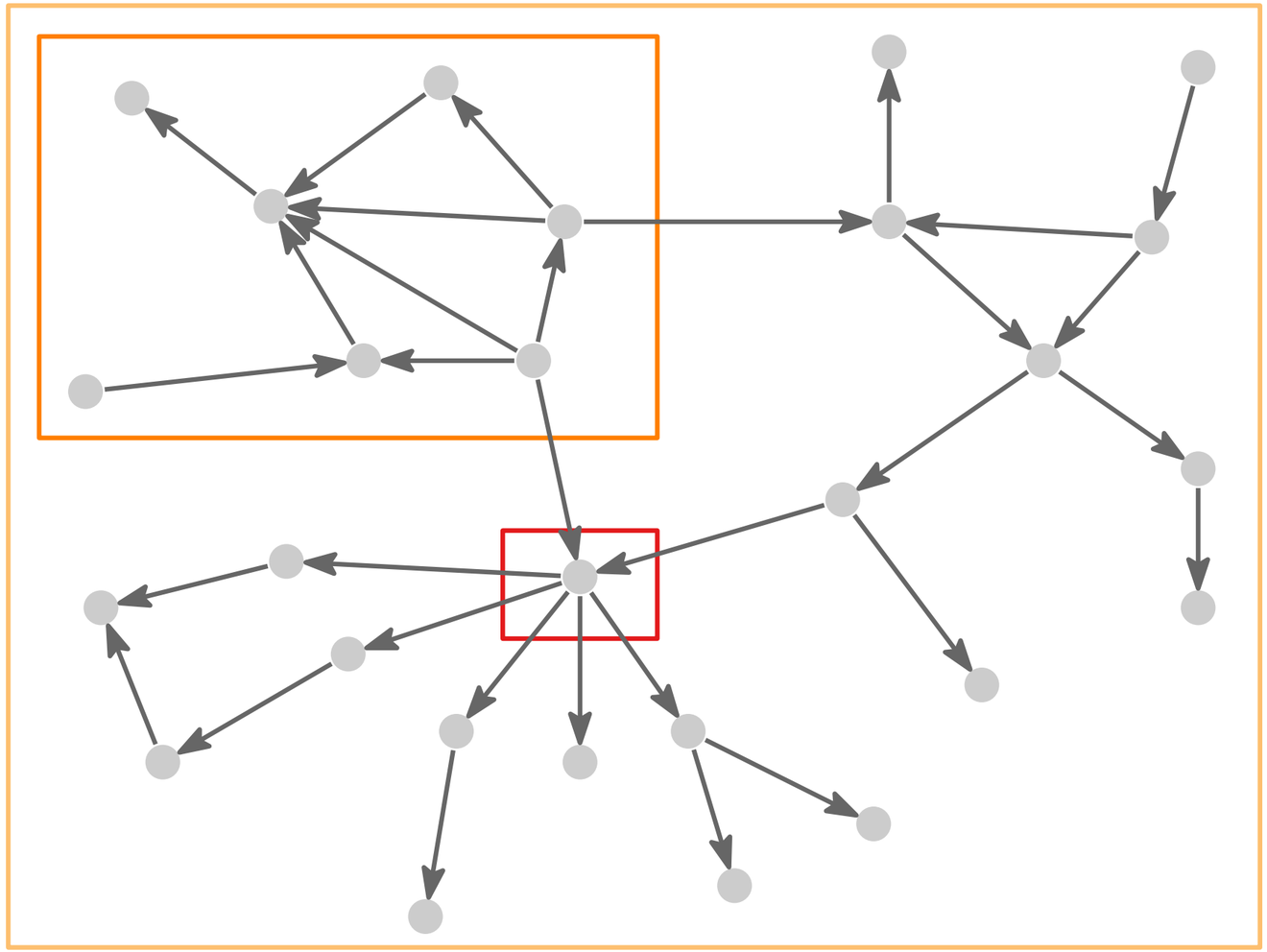
Micro Level

Nodes & Neighborhoods

Meso Level

Motifs & Communities

Basic Levels of Analysis



Micro Level

Nodes & Neighborhoods

Meso Level

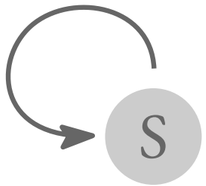
Motifs & Communities

Macro Level

Statistics & Invariants

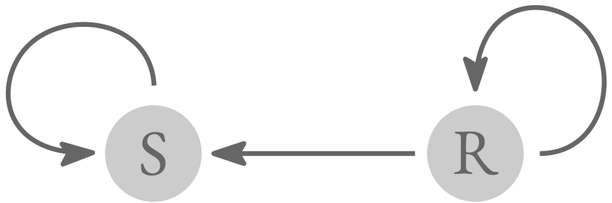
Types of Legal Documents

Types of Legal Documents



S Statutes

Types of Legal Documents



S Statutes

R Regulations

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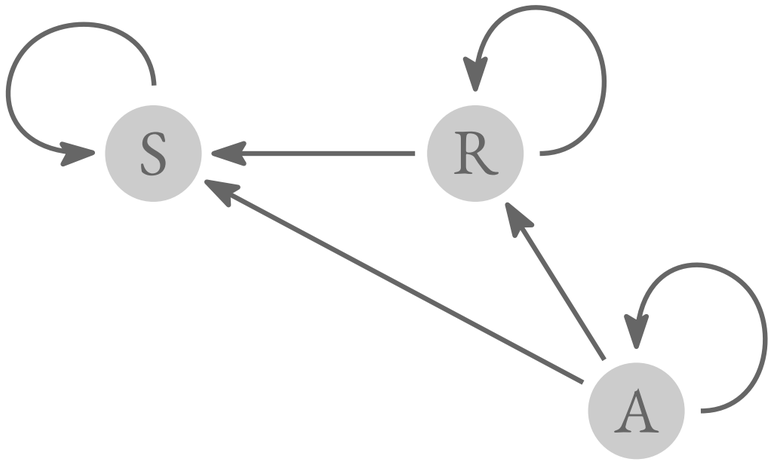

S Statutes

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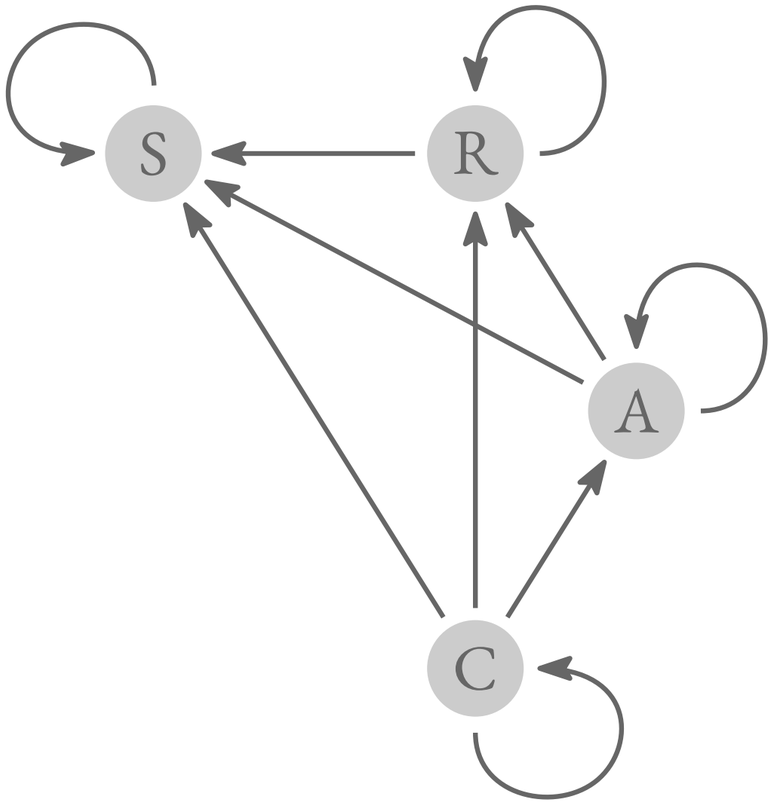
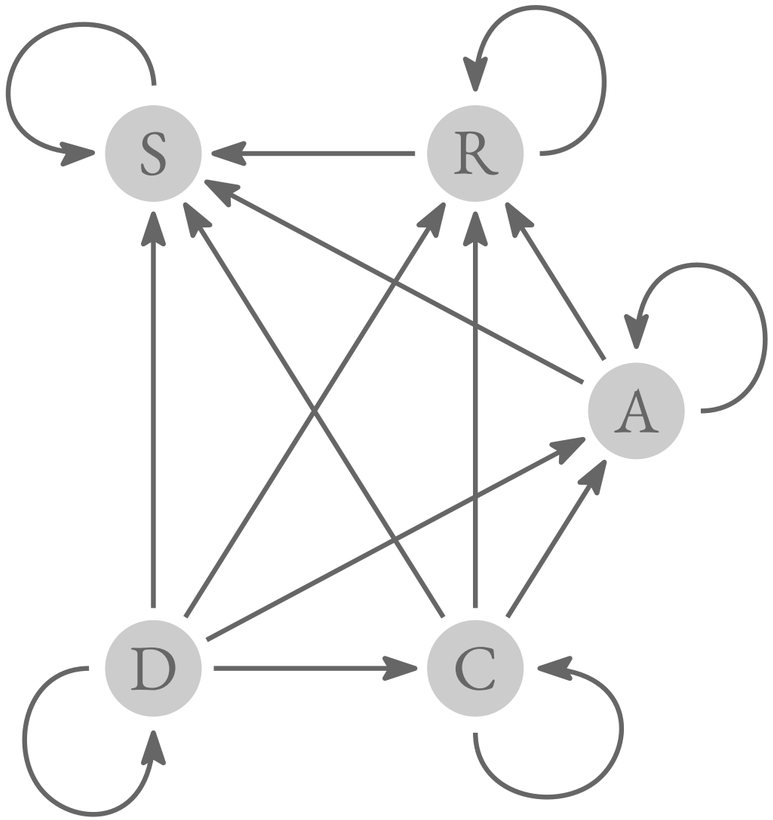
A Administrative Acts

Types of Legal Documents



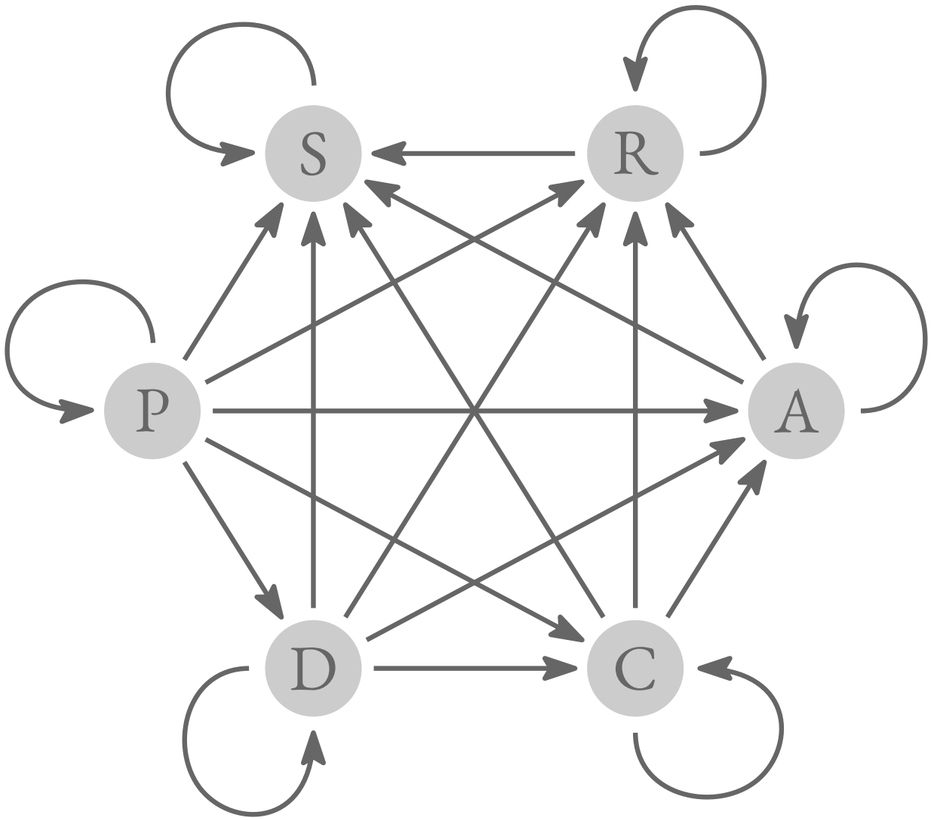
- S Statutes
- R Regulations
- A Administrative Acts
- C Contracts

Types of Legal Documents



- S Statutes
- R Regulations
- A Administrative Acts
- C Contracts
- D Judicial Decisions

Types of Legal Documents



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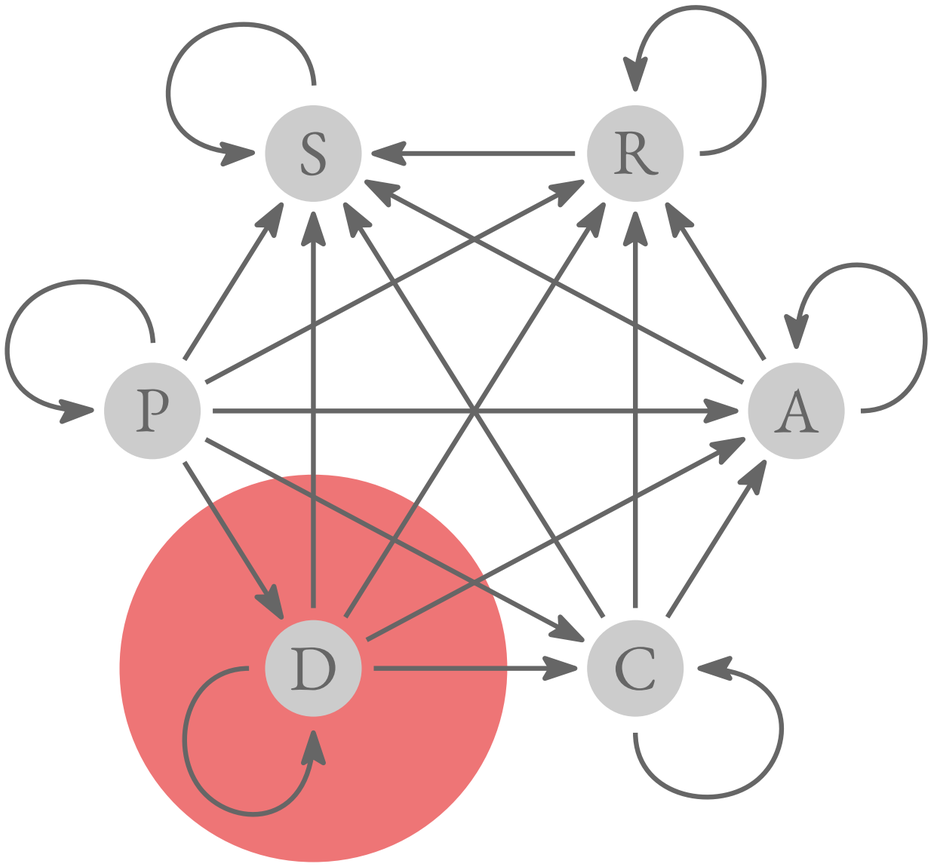
- P Scholarly Papers

Types of Legal Documents



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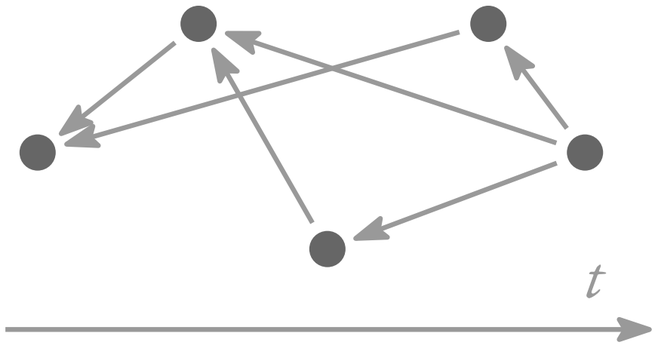
Judicial Citation Networks

Judicial Citation Networks

Traditional View



Nodes = Decisions

Edges = Citations

DAG-like

Analyzed as Aggregate

Judicial Citation Networks

BVerfGE 120, 274, Para. 271

aa) Covert surveillance measures by the state must respect an inviolable core of private life protected under Art. 1(1) of the Basic Law (cf. BVerfGE 6, 32 <41>; 27, 1 <6>; 32, 373 <378 and 379>; 34, 238 <245; 80, 367 <373>; 109, 279 <313>; 113, 348 <390>). Even overriding public interests cannot justify an interference with this core (cf. BVerfGE 34, 238 <245>; 109, 279 <313>). The development of one's personality within the core of private life encompasses the possibility of expressing internal processes such as emotions and feelings, as well as reflections, views and experiences of a highly personal nature, without fear of surveillance by state authorities (cf. BVerfGE 109, 279 <314>).

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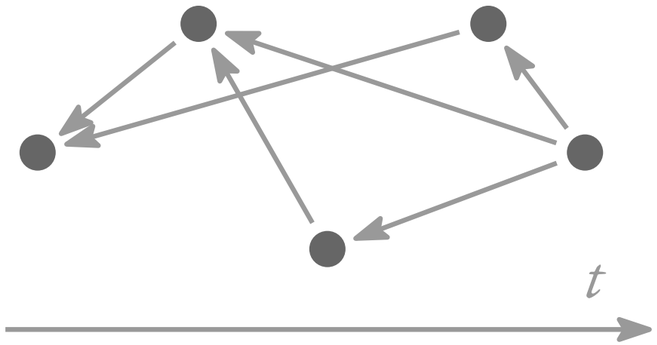

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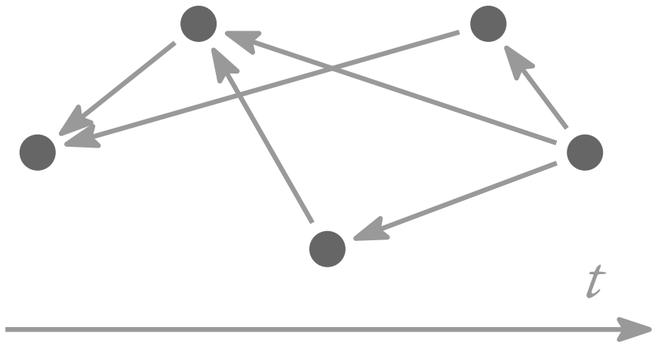

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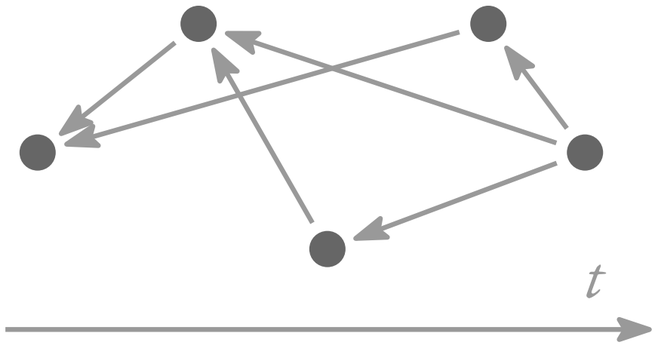

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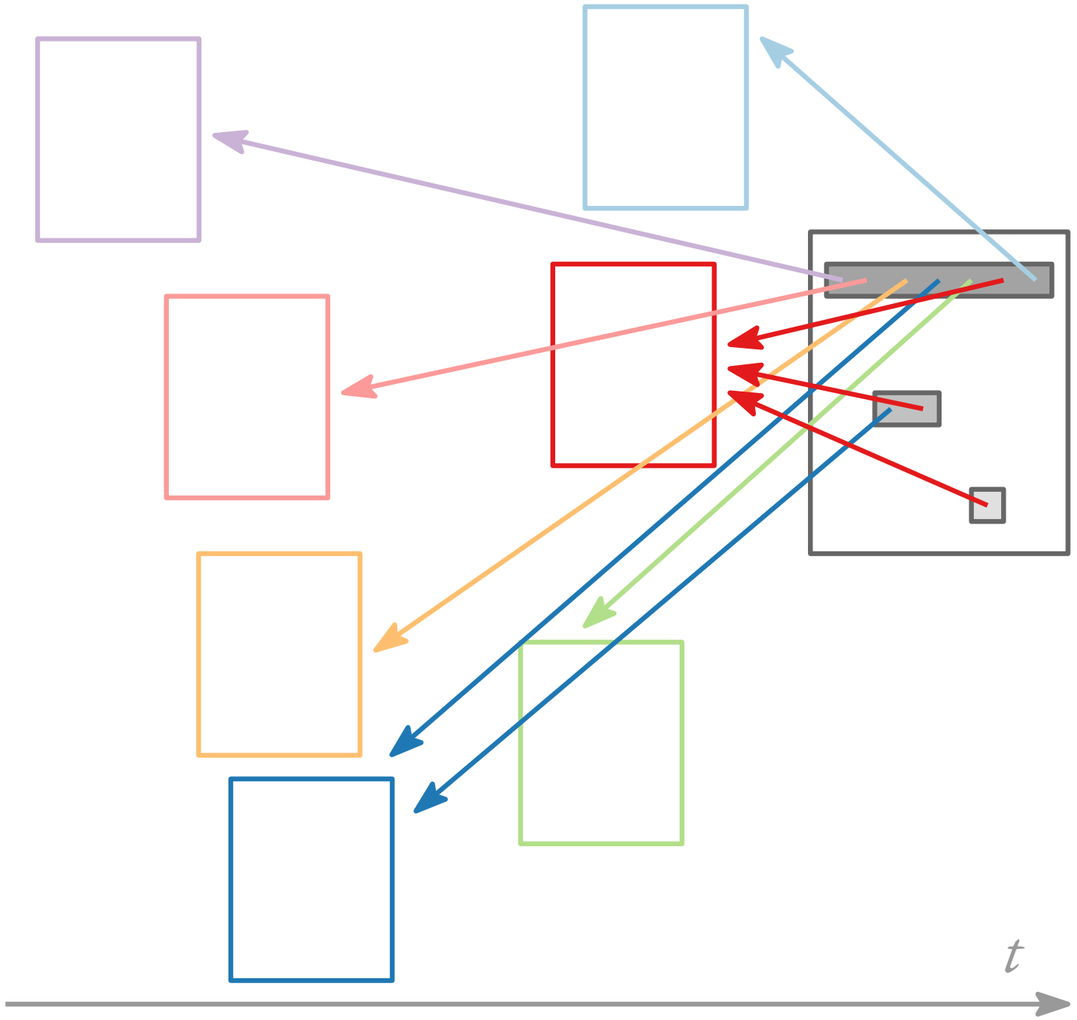
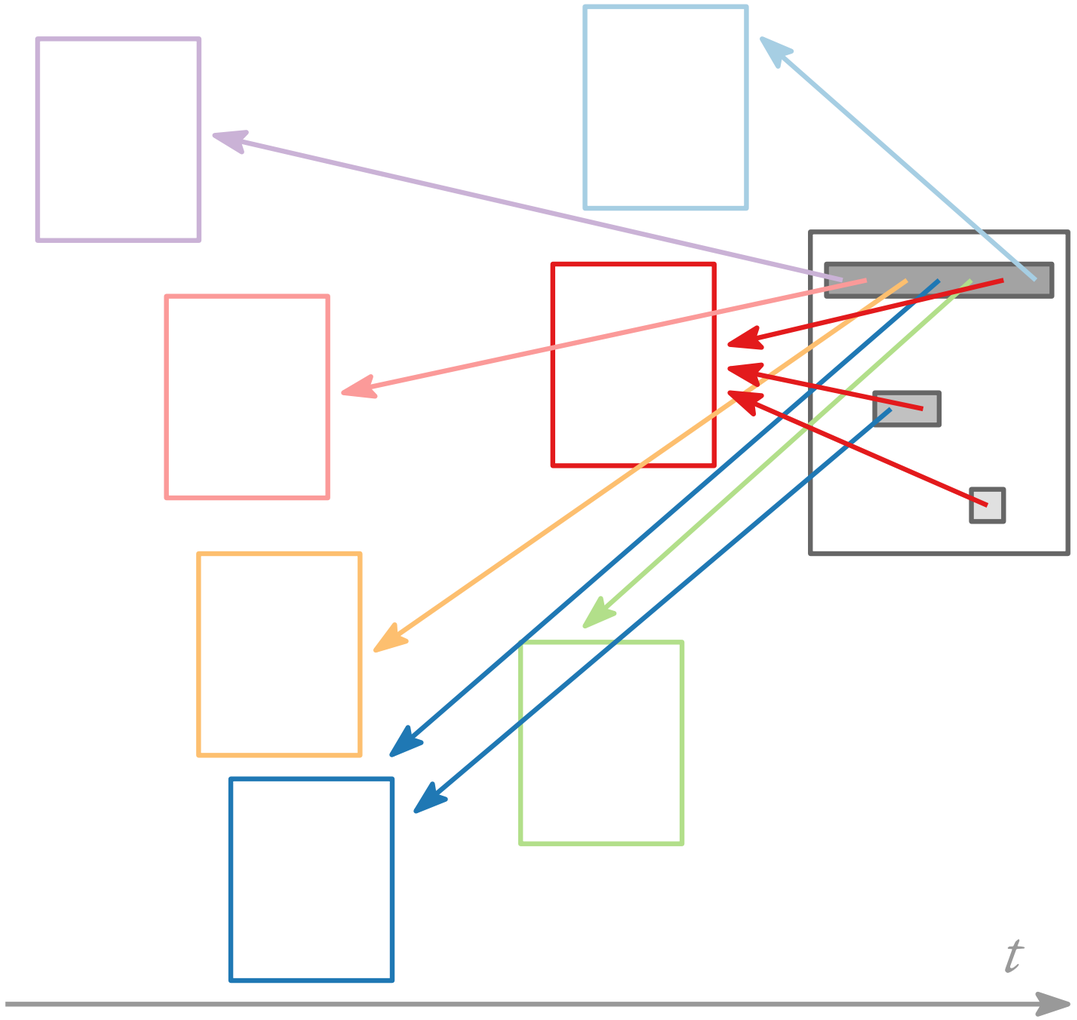


Judicial Citation Networks

BVerfGE 120, 274, Para. 271

aa) Covert surveillance measures by the state must respect an inviolable core of private life protected under Art. 1(1) of the Basic Law (cf. BVerfGE 6, 32 <41>; 27, 1 <6>; 32, 373 <378 and 379>; 34, 238 <245; 80, 367 <373>; 109, 279 <313>; 113, 348 <390>). Even overriding public interests cannot justify an interference with this core (cf. BVerfGE 34, 238 <245>; 109, 279 <313>). The development of one's personality within the core of private life encompasses the possibility of expressing internal processes such as emotions and feelings, as well as reflections, views and experiences of a highly personal nature, without fear of surveillance by state authorities (cf. BVerfGE 109, 279 <314>).

"Doubly temporal" (directed) hypergraph

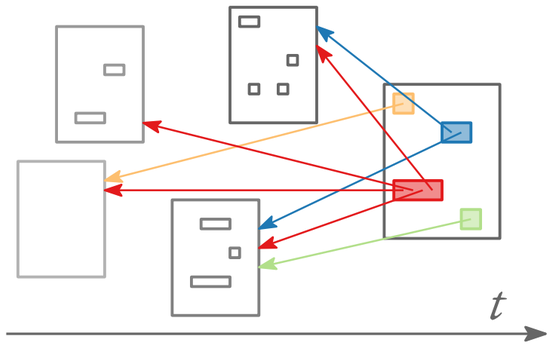


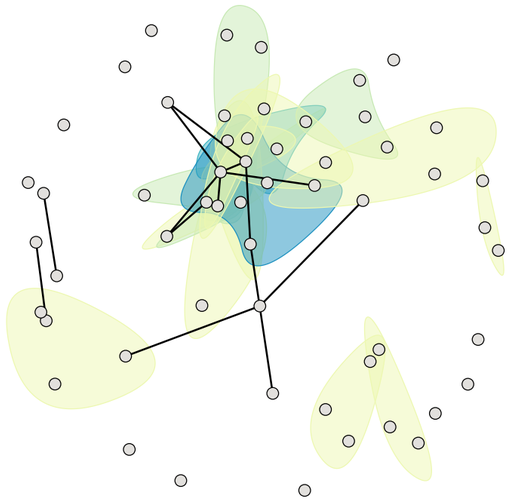
Judicial Citation Networks

Nodes = cited decisions

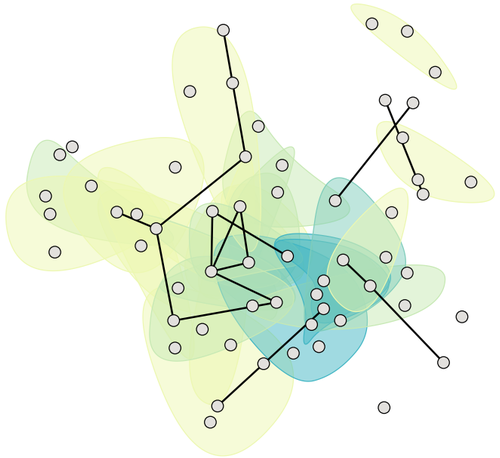
(Hyper)edges = citation blocks

No depiction of time

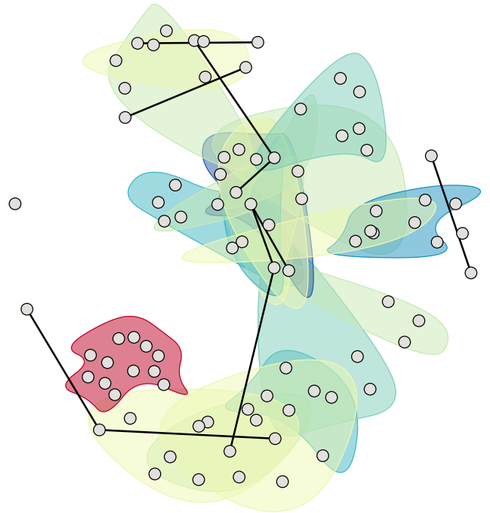




BVerfGE 125, 260—Retention of Data



BVerfGE 153, 317—ESM Treaty



BVerfGE 153, 1—Headscarf III

Part II

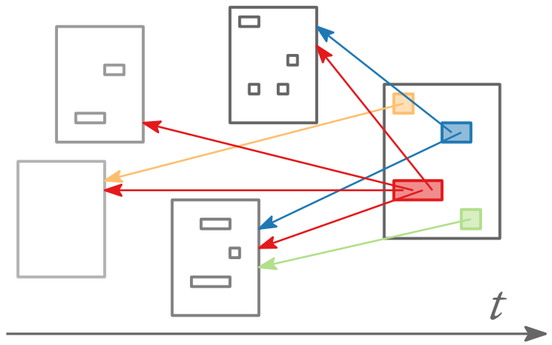
The Anatomy of Legal Argumentation

WIP
Comments Welcome

Part II

The Anatomy of Legal Argumentation

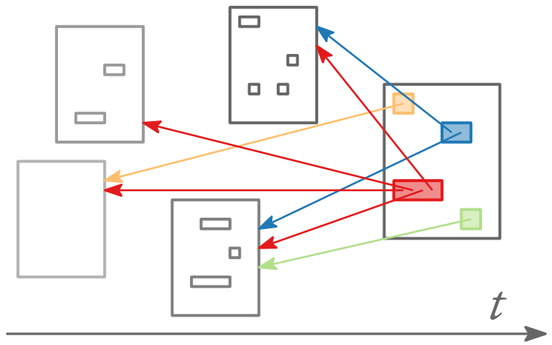
Motivating Questions



Motivating Questions

When, how, and why do courts use precedent in their decisions?

→ Test expectations and narratives from legal theory



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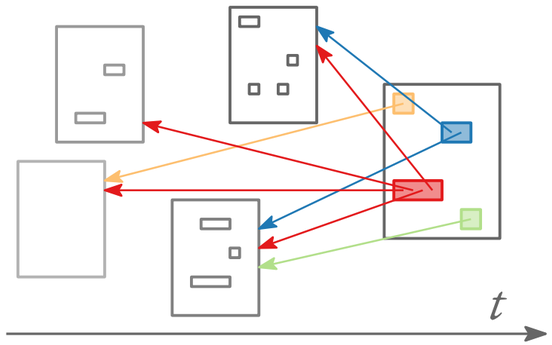

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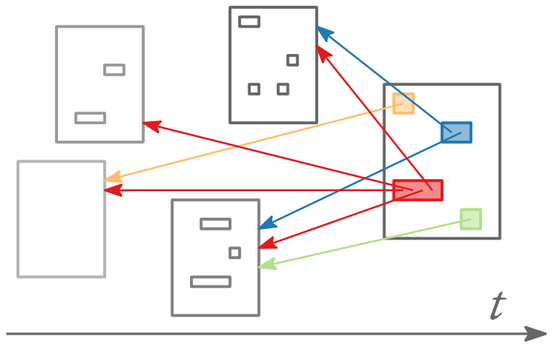

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→ Need new methods to leverage information contained in special structure

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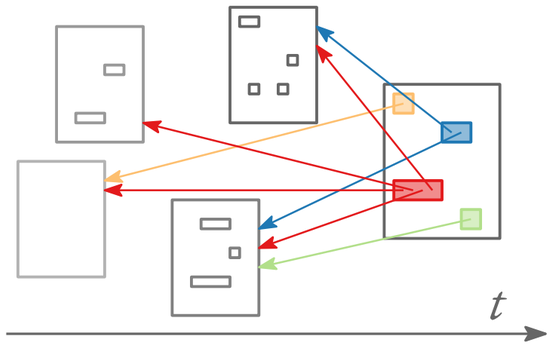

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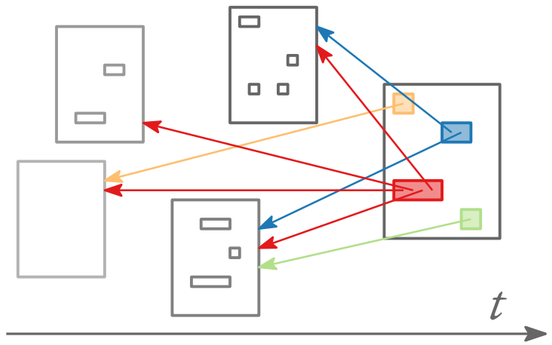
NB: Structure and dynamics of precedent citations $\subsetneq$ legal argumentation (→ Outlook)

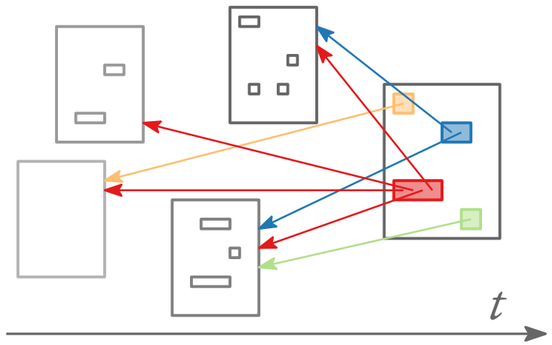
Legal Data



Legal Data

German Federal Constitutional Court's official collection

70+ years of constitutional jurisprudence (1951–2022)



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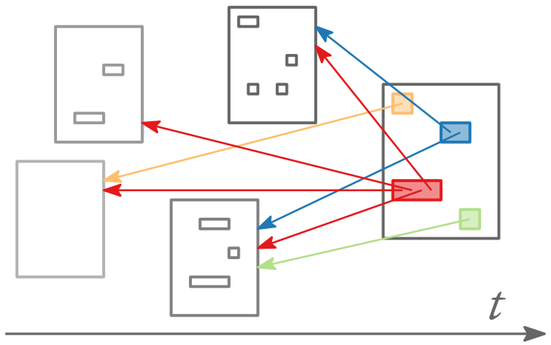
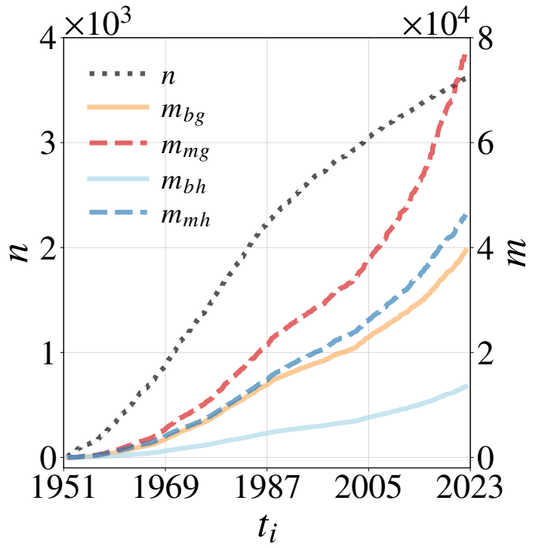
Basic aggregate statistics (subject to modeling choices):





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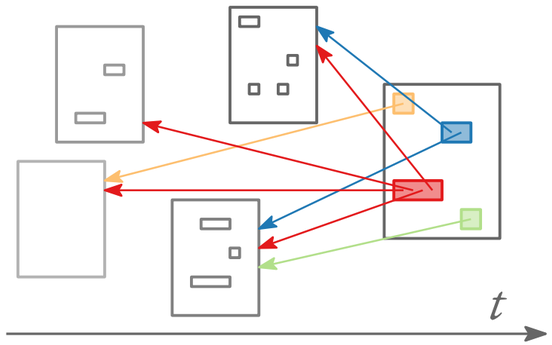

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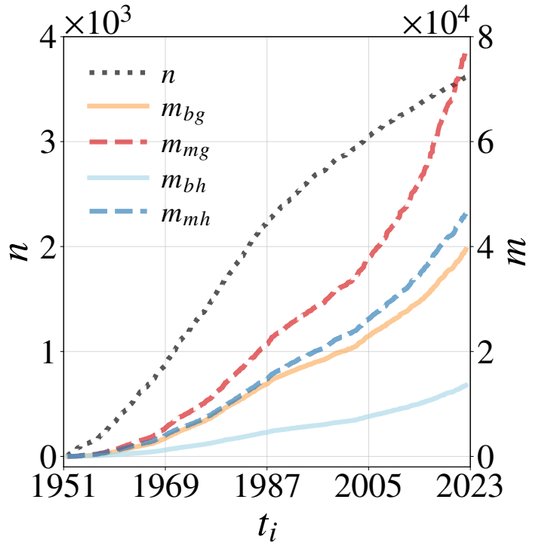

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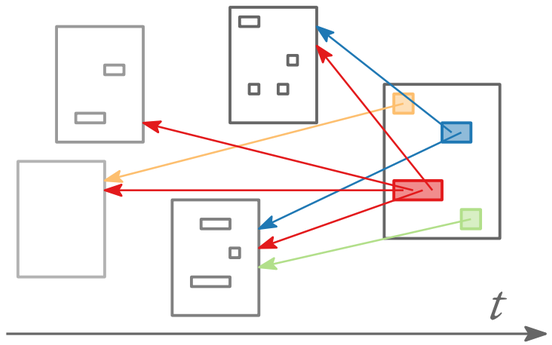

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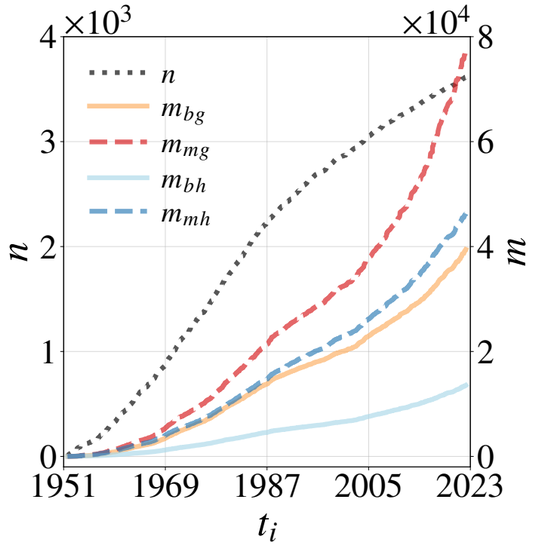
- 39 390 unique citations (source, target)



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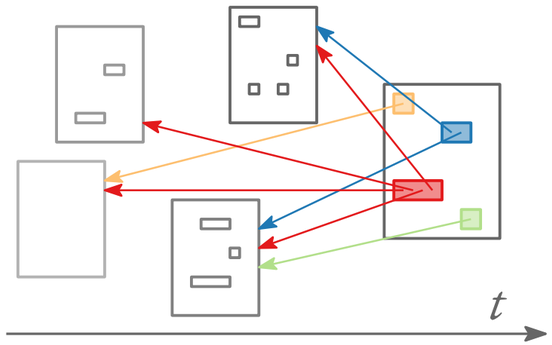

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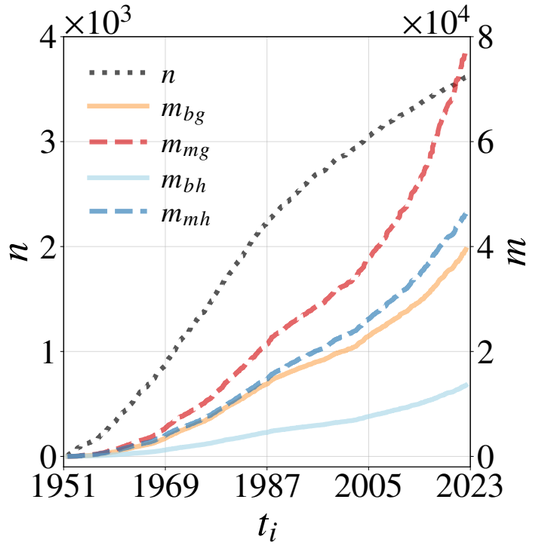

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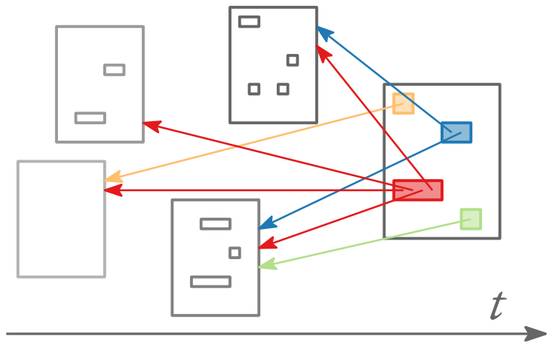

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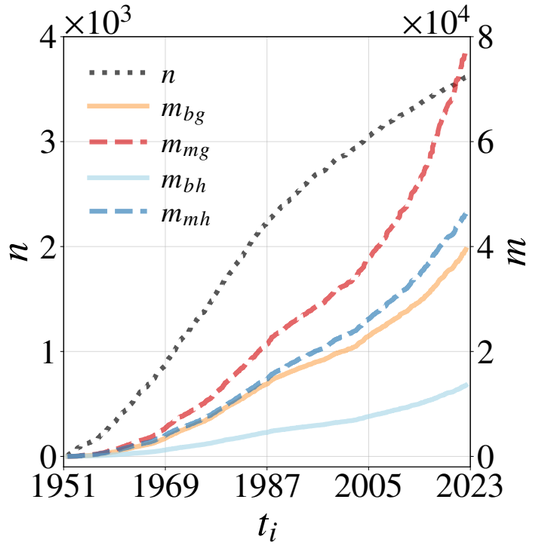

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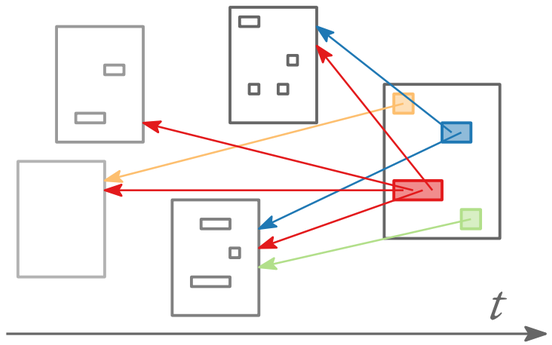

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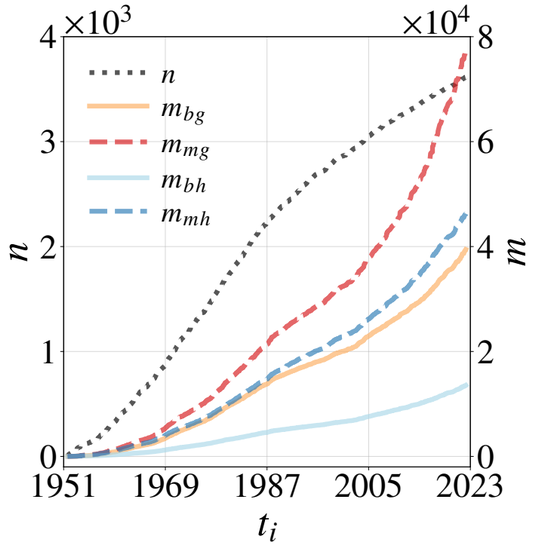
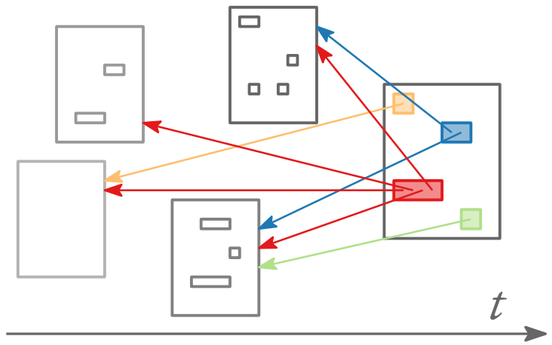
- 77 246 unique citation events (source, target, citation block)
- 46 219 citation blocks
- 13 571 unique citation blocks (target sets = hyperedges)

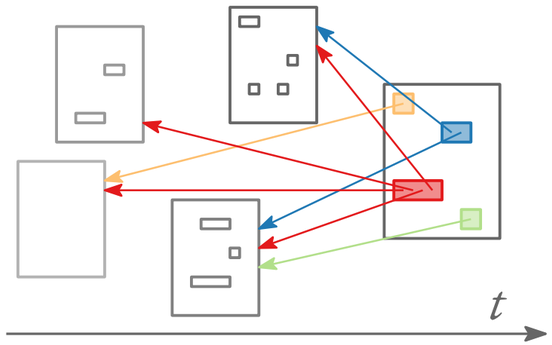


Network Representation



Network Representation

Each decision is represented as. . .



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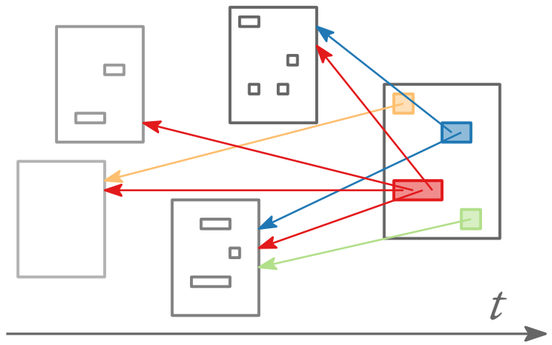

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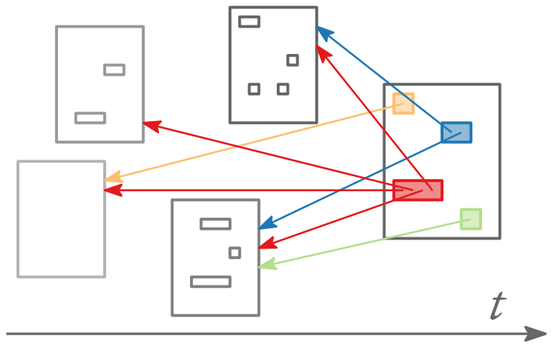

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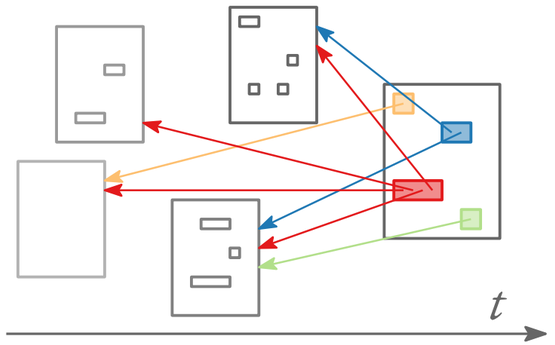

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→ self-referential!

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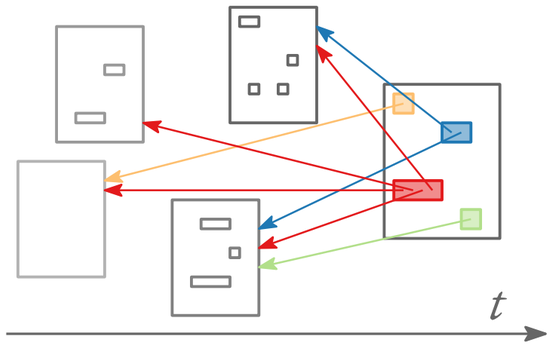

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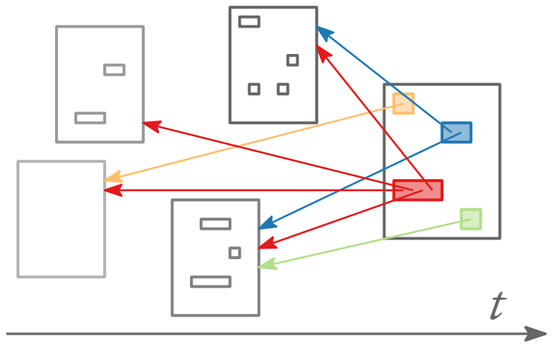

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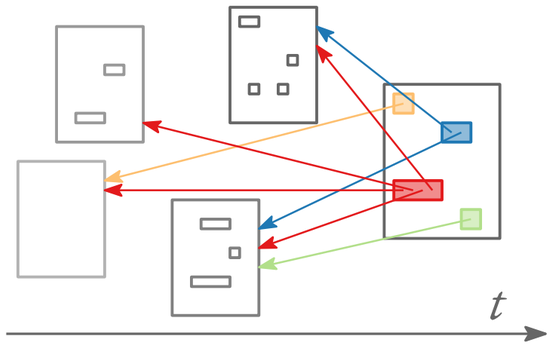

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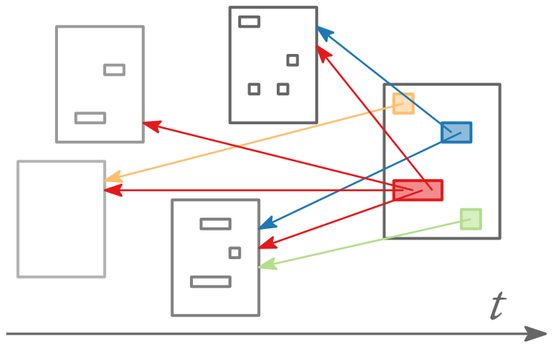

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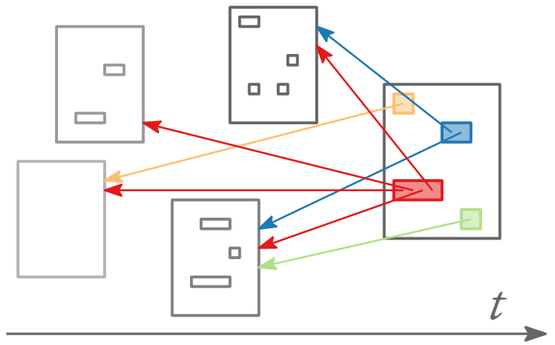

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How can these order structures help us trace doctrinal development?

Lines of Jurisprudence

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≈ Gradual development of doctrine in a sequence of decisions

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$t_0 : A, t_1 : B \to A, t_2 : C \to BA, t_3 : D \to CBA, \ldots$ (linear growth)

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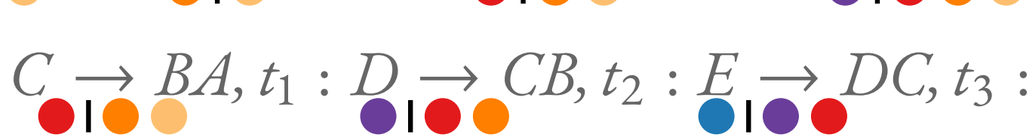

$t_0 : C \to BA, t_1 : D \to CB, t_2 : E \to DC, t_3 : F \to ED, \ldots$ (*n* most recent)

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Lines of Jurisprudence

≈ Gradual development of doctrine in a sequence of decisions

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These cannot be analyzed with traditional graph representations!

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$t_0 : C \to BA, t_1 : D \to CB, t_2 : E \to DC, t_3 : F \to ED, \ldots$ (n most recent)

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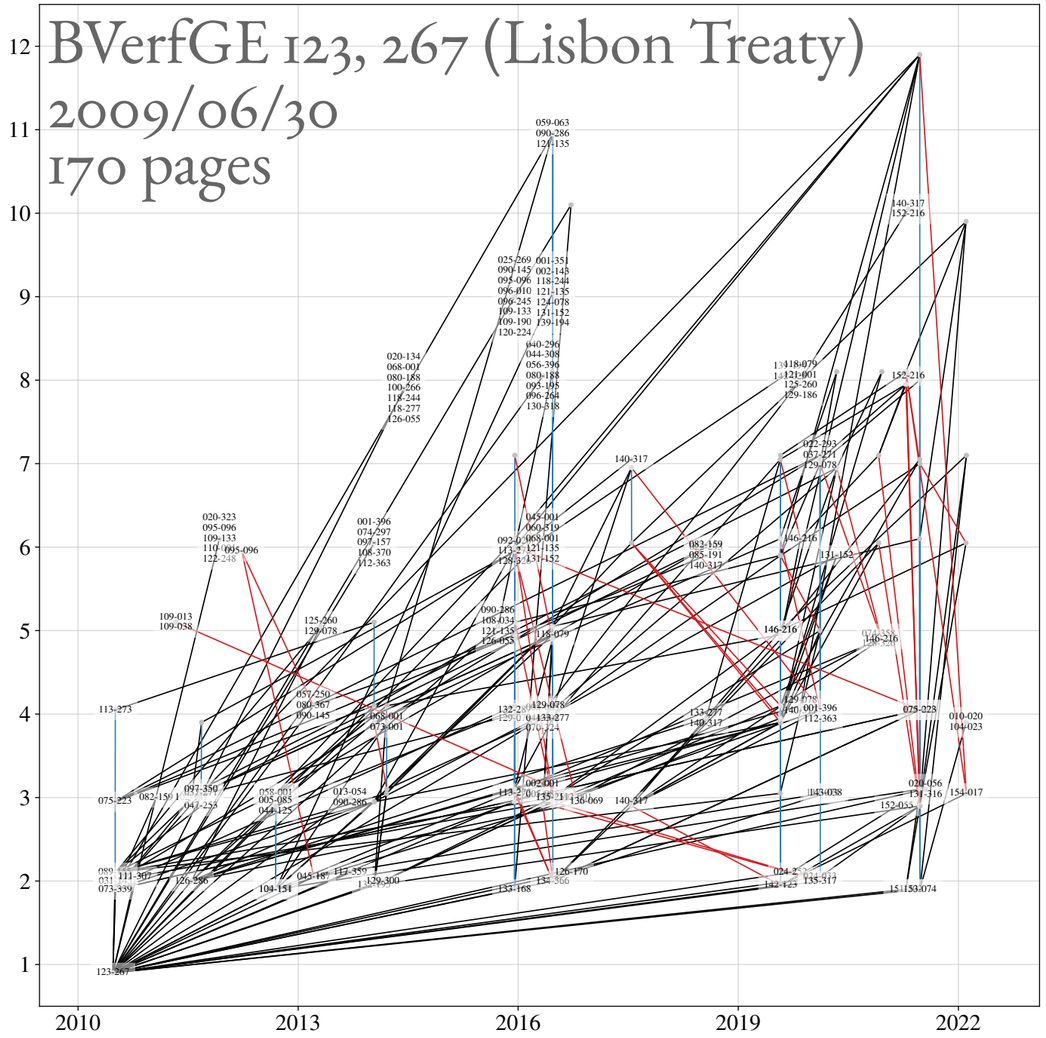
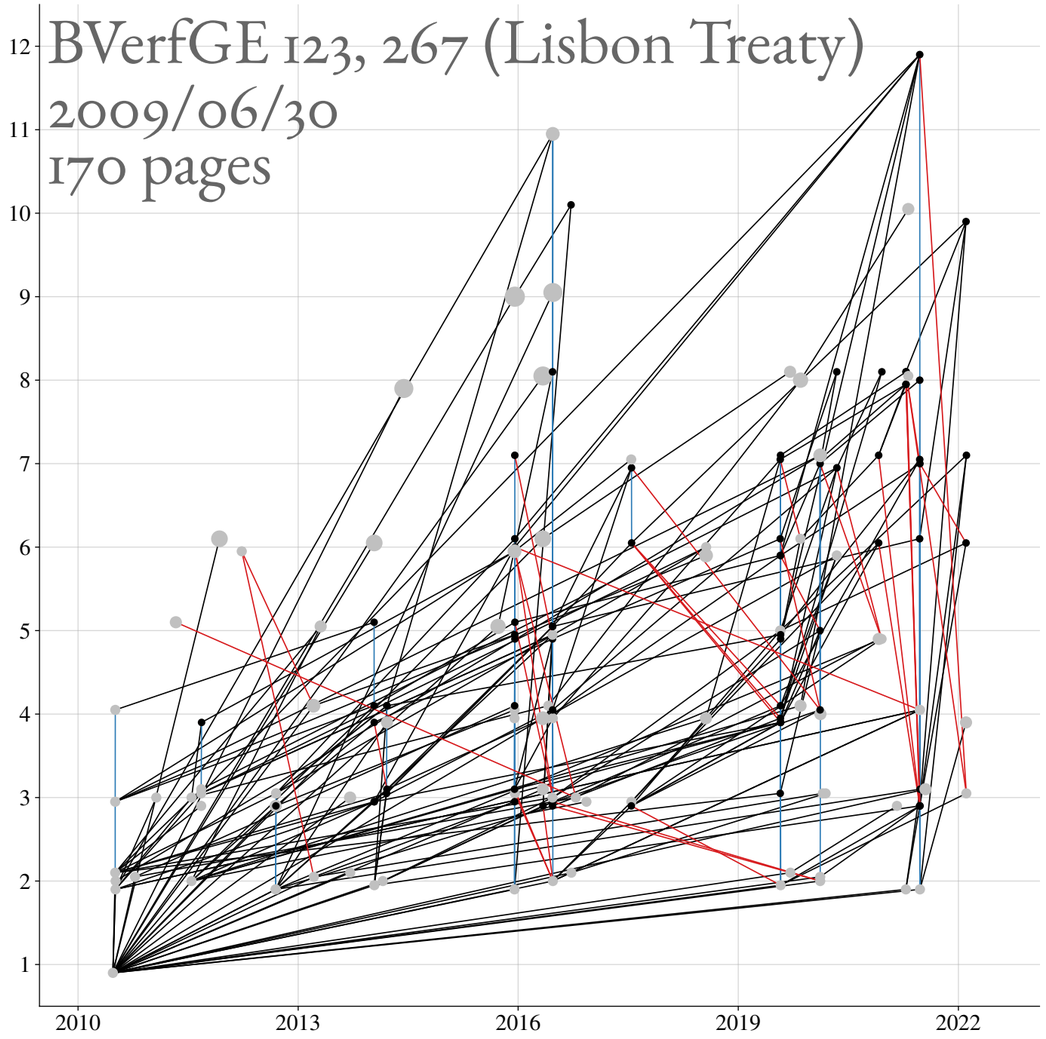
Visualizing Lines of Decisions



- Forward inclusions
- Sideward inclusions
- Backward inclusions

Visualizing Lines of Decisions



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- Perfect joins
- Additions (potentially with joins)

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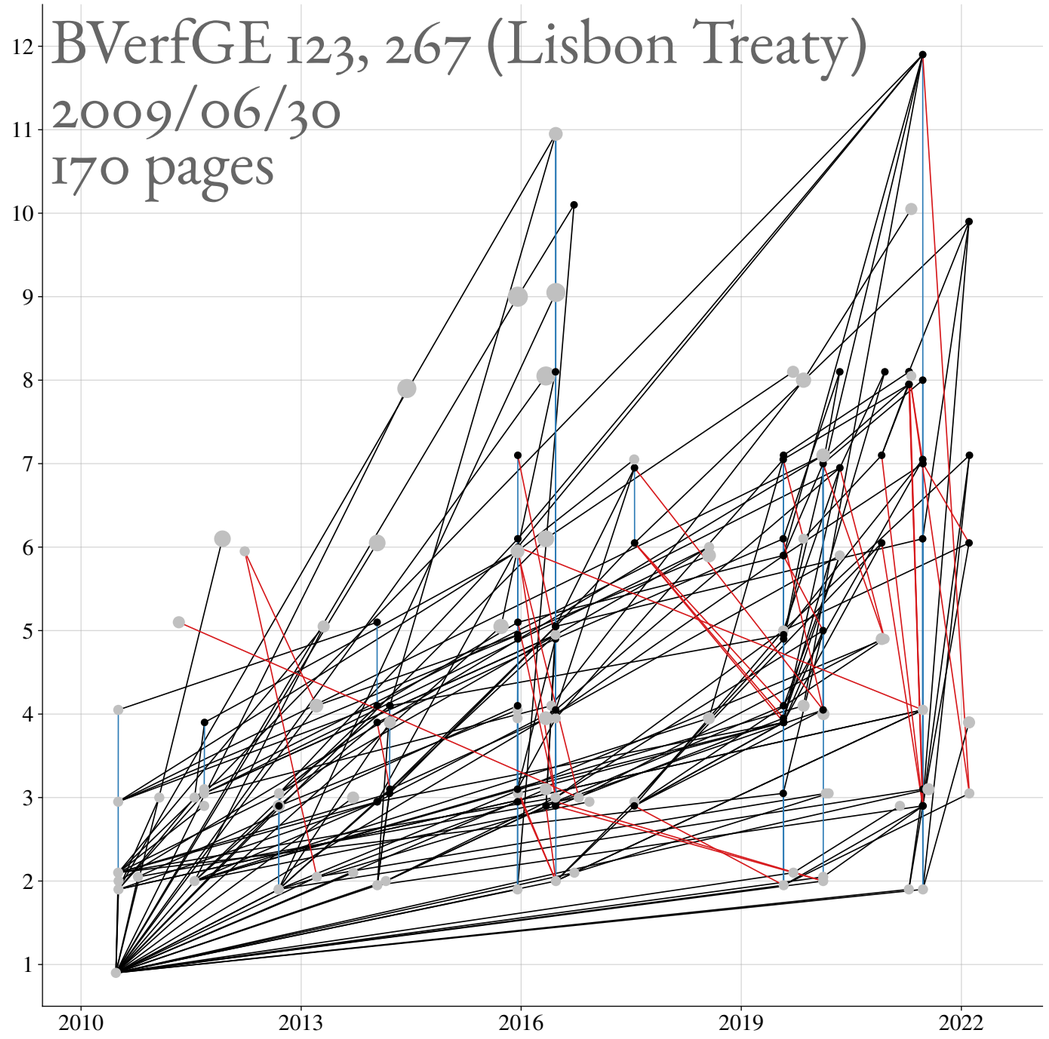


– Forward inclusions

– Sideward inclusions

– Backward inclusions

• Perfect joins

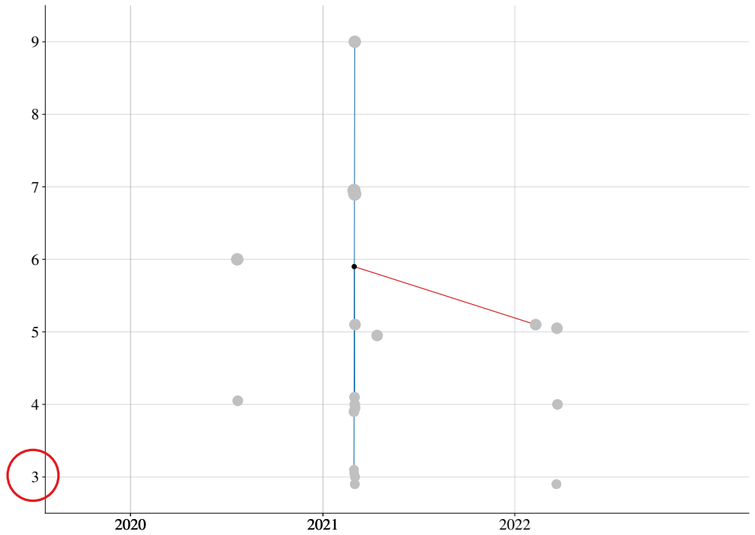
• Additions (potentially with joins)

NB: If this was interactive, we could further, e.g.,

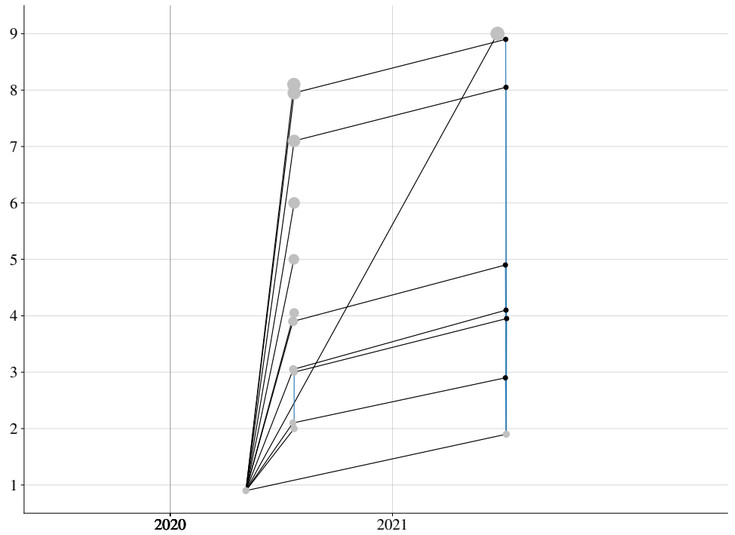
– filter hyperedges by source decision
– show timings, frequencies of hyperedge occurrences
– …

Visualizing Lines of Decisions

Varying integration of similarly-aged recent decisions



BVerfGE 150, 194
(Order, Organstreit Proceedings)
2018/12/11, 10 pages



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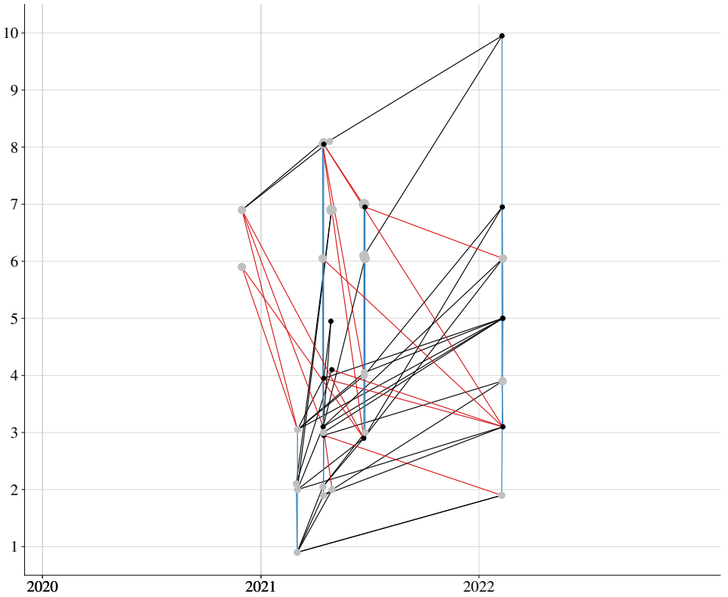

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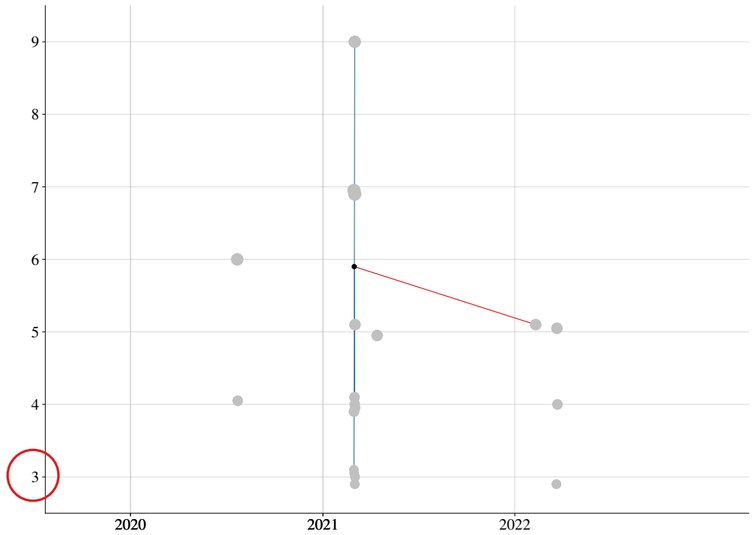
Visualizing Lines of Decisions

Varying integration of similarly-aged recent decisions

How can we formalize this?



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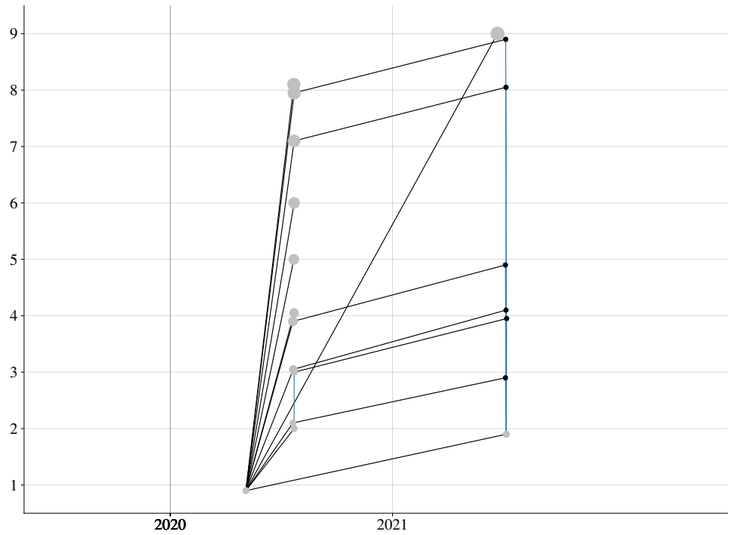

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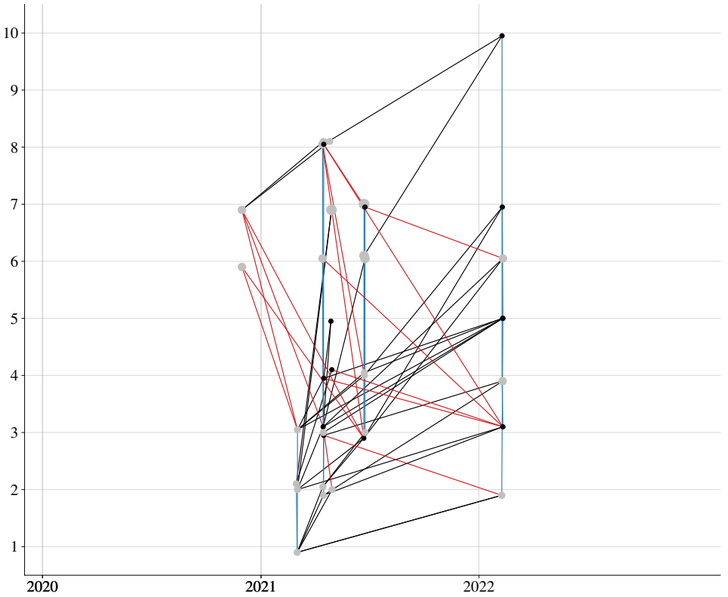

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Defining Decision Posets

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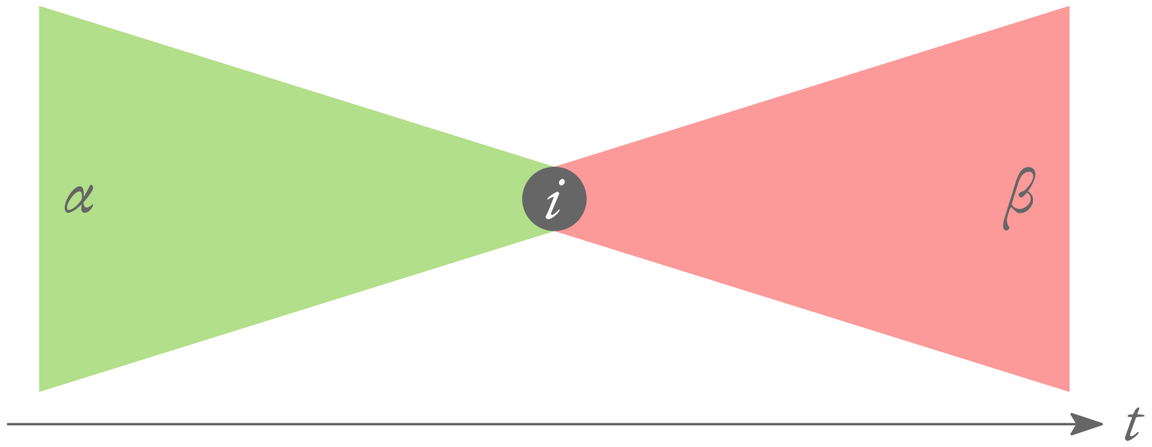

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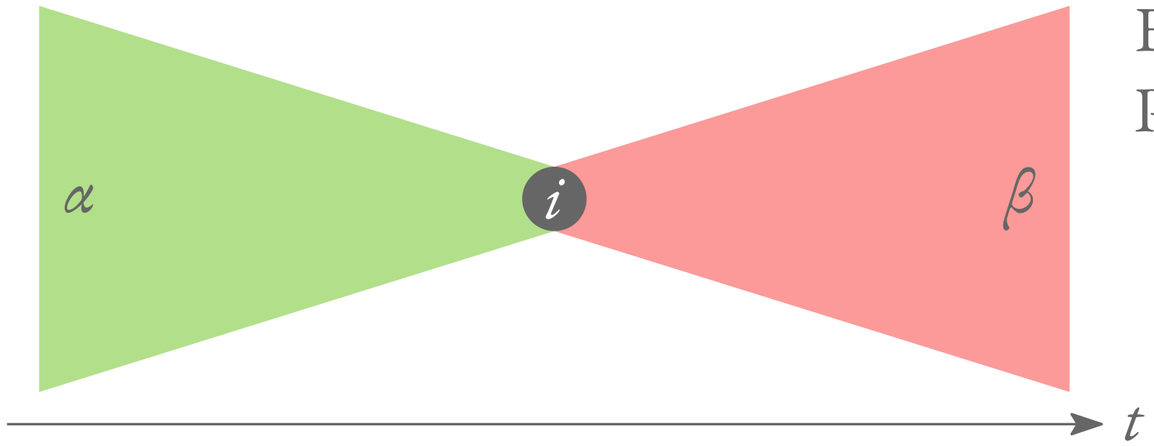

Earlier visualizations:
Passive 1-posets of selected decisions

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Formalizing and quantifying argumentation patterns based on the *entire* directed hypergraph

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How can we integrate domain studies with method development? → HSSH Brownbag 11/27

Thank you! Questions?

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